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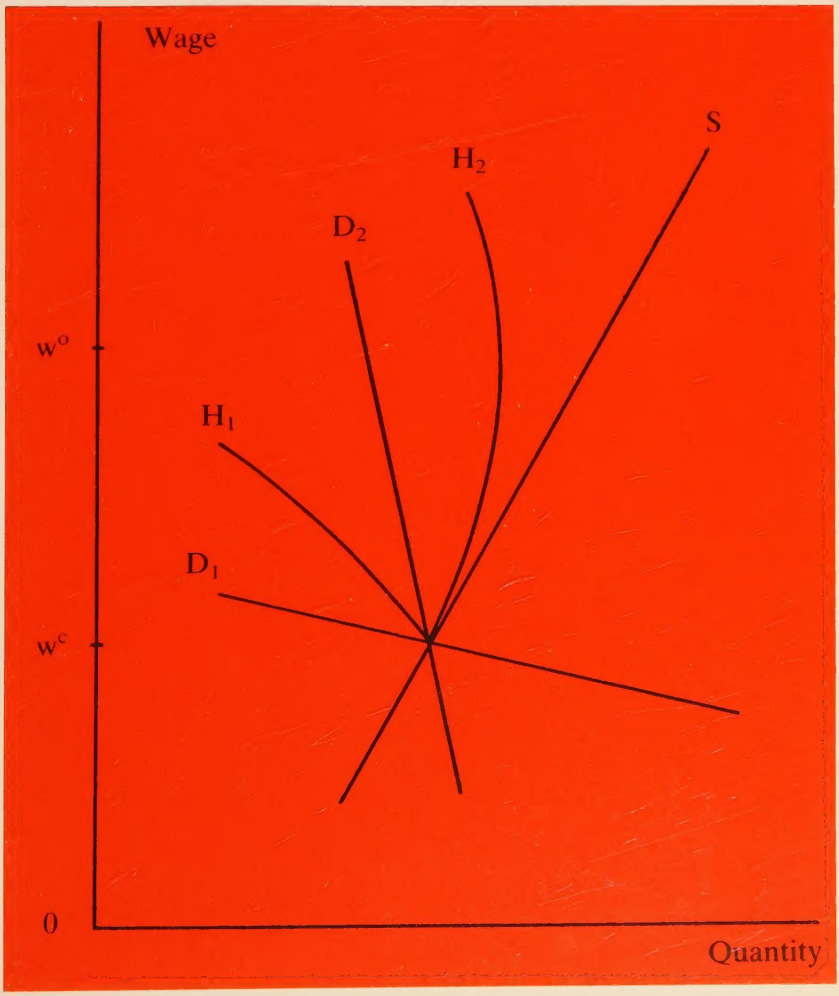
A Model of the Price Mechanism



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A Model of the Price Mechanism

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Abstract

This is an attempt to synthesize modern noncooperative decision theory and classical demand and supply analysis. Game theory is used to clarify the character of the decision problem facing price-setting firms. The concept of a homogeneous market is used to make the analysis tractable.

The main result is that independent firms choosing prices as well as quantities tend to quote that price which maximizes the industry's collective sales revenues for prices above the market-clearing level. The market price will consequently depend on the elasticity of the aggregate demand curve. Inelastic demand implies a noncompetitive equilibrium price for atomistic as well as oligopolistic industries. On one hand the oligopoly price will never approach the monopoly price as long as firms are free to produce what they like at the ruling market price. On the other hand price competition forcing the price down to the competitive price will never be rational for atomistic firms as long as they are free to change their prices at any time.

An analogous result holds in labour markets when workers have a purely instrumental attitude towards time spent in market activities, so that the division of this time between work and job search does not matter in itself, only the total payoff in terms of goods. For then every member of the worker's union will vote for that wage which maximizes the industry's wage bill. This wage is also supported by noncooperative behaviour when (reservation) wages are perfectly flexible.

Doctoral Dissertation
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Contents

Preface		6
Notation		7
Chapter 1	On decentralized price setting	9
Chapter 2	Noncooperative flexible pricing in a homogeneous market with Jörgen W. Weibull	21
Chapter 3	Oligopolistic pricing	27
Chapter 4	Noncompetitive equilibria in atomistic markets	44
Chapter 5	Job search and effective labour supply	57
Chapter 6	Effective labour supply and optimal wage setting	65
Notes		79
References		84

Preface

This is an attempt to synthesize modern noncooperative decision theory and classical demand and supply analysis. Game theory is used to clarify the character of the decision problem facing price-setting firms. The concept of a homogeneous market is used to make the analysis tractable.

The basic ideas were first presented in a seminar at the Swedish Institute for Social Research (SOFI) at the University of Stockholm in April 1984. Since then they have been successively developed in a sequence of independent papers that are here presented as chapters of a monograph for the simple reason that they do constitute variations of a common theme. Chapter 1 introduces the theme through some simple examples and summarizes the monograph and its resultant vision of the price mechanism.

As made clear by the specific acknowledgements attached to each chapter I am indebted to many people for useful discussions and comments. Here I would like to single our four of them for special thanks. Having produced a first sketch in February 1984 it was greatly encouraging to learn that Jörgen W. Weibull had written a note (also dated February 1984) pointing in the same direction, as witnessed by chapter 2. Discussions with Jörgen in the spring of 1986 have also been very fruitful. As an external discussant Anders Borglin made some vital contributions, first in a seminar at SOFI in November 1985 and second in a seminar at the Department of Economics in May 1986. His visits were followed by hectic revisions for which the reader should be truly grateful.

Lars E.O. Svensson has read almost all of my preliminary drafts and commented extensively on them. His sharp and constructive criticism of my early attempts was an absolutely essential input in my work. I owe him a great deal. Eskil Wadensjö's reputation as an interested and efficient thesis adviser I can gratefully confirm. His helpful comments and thought-provoking references – from Bagge (1917) to Axelrod (1984) – have been indispensable. And without his unswerving support and discreet insistence the project of writing a thesis would certainly have been postponed yet another decade.

This work has been carried out at SOFI under ideal conditions. The institute's blend of disciplines is most stimulating. Its secretarial staff is competent and kind. I am particularly grateful to Anne-Maj Folmer-Hansen for transforming my rough drafts into neat manuscripts with skill and patience.

For twelve years, beginning in 1970, I was one of the editors of Häften för Kritiska Studier. I shall always remember those years with pride and gratitude. Any originality which may accrue to the following approach to price theory I owe to and wish to share with my friends from *häften*.

Stockholm, June 1986 *Ante Farm*

Notation

n: number of firms
p_i: prices
p: price if uniform ($p_i = p$)
q_i: outputs
Q: aggregate output, $\Sigma_i q_i$
α_i: market shares, equal to q_i/Q with proportional rationing
$c_i(q_i)$: cost of producing q_i
c: average and marginal costs if $c_i(q_i) = cq_i$
$s_i(p)$: competitive supplies at p
$S(p)$: aggregate competitive supply, $\Sigma_i s_i(p)$
r_i: sales
π_i: profits
σ_i: strategies
$D(p)$: aggregate demand at p
η: demand elasticity, $\eta = \eta(p) = -pD'(p)/D(p)$
p^c: competitive price, $D(p^c) = S(p^c)$
p^o: maximizes collective sales revenue $pD(p)$ for $p \geqslant p^c$

When discussing labour markets (cf. chapters 5 and 6) an analogous notation is used, with w (wages) replacing p, and h (hours) replacing q.

Chapter 1

On decentralized price setting*

1. Introduction

Fundamental to economics is the notion of an "equilibrium price" determined by "equality between supply and demand". Equilibrium is, implicitly or explicitly, defined as a stationary point of a price adjustment process in real time. Thus prices may deviate from the market-clearing price out of equilibrium, but according to "the law of supply and demand" the process should converge to the market-clearing price.[1] In equilibrium the market price must be such that supply equals demand, since if it is such that supply exceeds demand there will be unsold goods and sellers will lower prices to get rid of them, and if demand exceeds supply, unsatisfied buyers will bid up the prices, as every textbook tells every new student in economics.

An immediate corollary of this story is that firms with perfect knowledge of the industry's demand and supply curves should leap instantly to market-clearing price offers, as emphasized already by Marshall (1920 p. 278) and Chamberlin (1962 p. 26). It is also easy to see that a market-price *less* than the market-clearing price cannot be supported by profit-maximizing firms – which excludes excess demand from equilibrium.But proving incompatibility of excess supply and self-interest is by no means a trivial task. Realizing this I attempted to do it. When I failed I began writing this book.

The problem tackled in what follows is in other words to depict firms' pricing and production as rational decisions conditional on perfect information on the industry's demand and supply curves. Our attempt to solve it will be based on game theory. The basic ideas are introduced in this introductory chapter through some simple examples, where a firm's decision variable is *either* quantity *or* price. (Simultaneous price and quantity adjustment will be analysed in subsequent chapters.)

In an auction market the only decision variable available to a firm is its output. Rational output decisions and the resultant (noncooperative) equilibrium price are derived in section 2. In section 3 we discuss Marshall's corn-market example, where equilibrium is established through pure price adjustment. In some markets sales take place before production. Thus the market process begins with firms announcing prices and consumers placing orders with the lowest bidder(s). Then firms produce

what has been ordered from them. This market structure (production to orders) is analysed in section 4.Results from sections 2-4 are summarized in section 5, emphasizing the differences between the three examples. The present chapter ends with a summary of the following chapters.

There is no Walrasian auctioneer in the present model of the price mechanism but there is a *statistician*, recording and announcing aggregate information about market conditions. In practice the statistician may be a central bureau of statistics or a trade association or a "barometric" price leader, which "commands adherence of rivals to his price only because, and to the extent that, his price *reflects market conditions* with tolerable promptness." (Stigler 1968 p. 228. My italics.)

If firms' price and output decisions do "reflect market conditions with tolerable promptness", they will not contradict the estimates of the market demand and supply curves upon which they are based. Then the market will be in a stationary state. Otherwise the firms' theory of predicting aggregate demand and supply curves must be revised, but the details of this (econometrical) learning process is outside the scope of the present study. It must be emphasized, however, that according to the present approach to price adjustment convergence to a stationary state involves learning the *aggregate* demand curve, not *individual* demand curves, the usual point of departure in the existing price-setting literature.

An individual demand curve is usually attributed to heterogeneity (Chamberlin 1962), disequilibrium (Arrow 1959) or uncertainty (Diamond 1971). The concept is often introduced in unorthodox and highly simplified models, as in the seminal paper by Hotelling (1929). But the concept can also be defined in a classical homogeneous market: taking his competitors' prices as given, an oligopolist's "contingent" demand function (conditional on these prices) is well-defined (Shubik 1959). On the other hand the derivation of individual (contingent) demand curves is a complex problem (Shubik 1959 ch. 5). And, as also emphasized by Shubik, imperfections like product differentiation only *adds to* this complexity. The prevalence of subjectively defined "perceived" or "conjectured" individual demand curves in the literature following Chamberlin (1962) and including Negishi (1960-61, 1979) and Hahn (1978) is therefore not surprising. In general firms can only guess at their individual demand curves.

A firm's individual demand curve is basically a *conditional function* given other firms' prices. This is most obvious in oligopoly theory. Consider, for instance, the customary payoff functions in differentiated products models,[2]

$$\pi_i(p) = p_i D_i(p) - c_i(D_i(p)),\ i = 1,2,...n,$$

defining a noncooperative game in prices for n firms. Here all that a firm takes as given is its competitors' *profit functions* (not their prices). But to

be able to solve this problem in the way game theory suggests a firm must know individual demand D_i as an explicit function of every price,

$$D_i = D_i(p) \text{ where } p = (p_1, p_2, \ldots p_n),$$

and not as an individual demand function of its own price where other prices appear as parameters only. Subjective perceptions of such complicated dependencies stretches the imagination.

2. *Competitive pricing in auction markets*

In the theory of perfect competition a firm is assumed to be so small that it "cannot influence the price of what it sells". It is a "pricetaker" and assumes that it can sell everything produced at the "given" price. In equilibrium the price "must" be determined by equality between demand and supply. In this section we shall rationalize this story without relying on a fictitious (Walrasian) auctioneer who simply proclaims the market-clearing price after a process of "tatonnement". Instead we shall postulate a *real* auctioneer.

In an auction market the only decision variable available to a firm is its output (q_i). Having completed production firms take their outputs to an auction market which determines the "demand price" for "each amount of the commodity", as in Marshall's *Principles* (book 5 ch. 3) and also, implicitly, in Chamberlin (1962 ch. 1). Thus the price will be $D^{-1}(\Sigma_j q_j)$, where D^{-1} denotes the inverse of the aggregate demand function $D = D(p)$. If $q_i/\Sigma_j q_j$ is "small", a firm's influence on the market price through its output undoubtedly appears "negligible", but how can we conclude that firms will produce that aggregate output which corresponds to a market-clearing price?

In an attempt to reconstruct Chamberlin's argument for instantaneous convergence to competitive equilibrium when firms have perfect information on the aggregate demand and supply curves (1962 p. 26) we first recall that competitive behaviour is characterized by 1) postulating an expected market price p^e and 2) assuming that everything supplied can be sold at this price. Then supply will be $S(p^e)$ and actual price $D^{-1}(S(p^e))$ *in an auction market*. With these assumptions there will be *expectational equilibrium* if and only if

$$D^{-1}(S(p^e)) = p^e.$$

The (unique) solution of this equation is obviously

$$p^e = p^c,$$

where p^c clears the market, $D(p^c) = S(p^c)$. Given perfect knowledge of demand and supply curves we interpret $p^e = p^c$ as *rational expectations* of the market price.

Moreover, with *imperfect information* we might interpret competitive

behaviour (appropriately modified below) as a *learning strategy* in atomistic auction markets in the following way. Suppose – after a shift of the demand curve – that firms are at a loss for which price to expect. But somehow – even when price expectations differ – they produce some amount Q_1 and observe $p_1 = D^{-1}(Q_1)$. This yields one observation on the demand curve. One more observation (Q_2,p_2) would make it possible to construct a linear approximation of the demand curve. But if Q_2 is more or less randomly chosen, as part of a trial-and-error process, no information on the new market-clearing price is obtained. With competitive behaviour, on the other hand, taking p_1 as given and producing $Q_2 = S(p_1)$, firms know that (p_1,p_2) will bracket p^c (due to the cobweb oscillations). Hence $p_3 = (p_1 + p_2)/2$ is certainly a better estimate of p^c than p_2 (which represents static expectations). Moreover, bringing $S(p_3)$ to an auction market will yield p_4 such that (p_3,p_4) will bracket p^c, yielding $p_5 = (p_3 + p_4)/2$ as a new estimate. In this way convergence to expectational equilibrium will be speeded up (and obtain even if the supply curve is flatter than the demand curve).

Thus competitive behaviour in auction markets is undoubtedly *consistent* in the sense that it does lead to *expectational equilibrium* in the competitive state (p^c), either instantaneously (with perfect information) or after a "competitive learning process". But is it also *rational*? To check this we turn to game theory and a simple example (due to Radner 1980).

For simplicity we assume that every firm has the same supply price (c), independent of its volume of production (implying a horizontal industry supply curve), and consider the short run, where the number of firms (n) can be taken as given. We also postulate a linear demand function

$$p = a - bQ,\ a,b > 0.$$

Given outputs q_i the auction market price will thus be

$$p = a - b\Sigma_i q_i.$$

A competitive state (q_i^c) is defined by

$$\Sigma_i q_i^c = D(c) = (a - c)/b.$$

Will firms be in equilibrium in such a state? Or rather, following Luce and Raiffa (1957 p. 63), can a firm's *decision theory* be in equilibrium if it predicts such a state? To check this we let firm i take its rivals' outputs q_j^c, $j \neq i$, as given and consider his "contingent" profit function

$$\pi_i(q_i) = (a - b(D(c) - q_i^c + q_i))q_i - cq_i.$$

Since

$$\delta\pi_i/\delta q_i = 2b(q_i^c/2 - q_i)$$

we see that $\delta\pi_i/\delta q_i < 0$ at $q_i = q_i^c$ so output restriction (in fact to $q_i^c/2$) appears profitable to an individual producer predicting a competitive state – which consequently cannot be an equilibrium.

To derive a rational output decision we start from the profit functions

$$\pi_i = (a - b\Sigma_j q_j)q_i - cq_i,\ i = 1,2,...n.$$

Thus a producer's profit depends not only on his own decision (q_i) but on his rivals' decisions as well (q_j, $j \neq i$). Since output rates are set simultaneously, without collaboration or communication, the firms face a *non-cooperative game*. Since a player does not know a priori the decisions to be taken by others, each of them must arrive at a "composite decision" $(q_1^*,q_2^*,...q_n^*)$, including not only his own "rational" action q_i^* but also "rational" *predictions* of the other agents' actions q_j^*, $j \neq i$.

The Cournot-Nash equilibrium[3] is such a "composite decision". Its "rationality" and status as a fundamental solution concept in economic theory has been emphasized not least by Johansen (1982). In the present context $q^* = (q_1^*,q_2^*,...q_n^*)$ defines a Cournot equilibrium if, for every i, $q_i = q_i^*$ maximizes

$$\pi_i = (a - b(Q^* - q_i^* + q_i))q_i - cq_i,$$

where $Q^* = \Sigma_j q_j^*$. Assuming $a > c$ it is easily verified that the unique Cournot equilibrium is given by

$$q_i^* = D(c)/(n + 1),\ i = 1,2,...n.$$

Thus aggregate output (nq_i^*) will be somewhat lower and market price somewhat higher than D(c) and c, respectively, but with many firms ($n \to \infty$) the difference will be negligible. Given information on the aggregate demand function (a and b) and the industry structure (n and c), every firm will find it rational to behave "almost" competitively when there are many firms. But this result does not generalize from auction markets to markets where prices are set by firms, as we shall now see.

3. *Rational pricing when supply is perfectly inelastic*

To introduce the basic principles of decentralized price-setting it is natural to begin with Marshall's corn-market example (in book V, ch. 2), where stocks offered for sale are already in existence and equilibrium is established through pure price adjustment. Marshall simply takes a market-clearing price for granted in this case: "because if it were fixed on at the beginning, and adhered to throughout, it would exactly equate demand and supply ... and because every dealer who has a perfect knowledge of the circumstances of the market expects that price to be established." (1920 p. 278). But on closer inspection decentralized price-setting in a homogeneous market is not as simple as that, given perfect knowledge of the aggregate demand curve and the (vertical) aggregate supply curve.

Let Q denote aggregate (fixed) supply, $Q = \Sigma_i q_i$, and p^c the competitive price, $D(p^c) = Q$. To begin with we now assume that prices set at

the beginning of the market period cannot be changed until the beginning of the next period. In this case a seller's pricing strategy can be characterized by a single number (p_i) and it is obvious that (p_i) = (p) cannot be a Nash equilibrium if $p > p^c$. For taking $p_j = p$, $j \neq i$, as given, a slight price decrease $p_i < p$ will yield a discontinuous increase in individual sales. So far the usual competitive story makes sense.

But we cannot conclude from this argument that (p_i) = (p^c) is a Nash equilibrium unless we assume that there always exists a Nash equilibrium. Now, existence of Nash equilibria is something which certainly cannot be taken for granted.[4] The present set-up is no exception. To see this let i take $p_j = p^c$, $j \neq i$, as given and contemplate a price $p_i > p^c$. Then consumers will buy $Q - q_i$ (but no more) from $n - 1$ sellers at price p^c and the "contingent" demand $x_i(p_i)$ from i at price $p_i \geq p^c$. $x_i(p^c) = q_i$ and we assume (without restriction) that x_i is differentiable. Then individual revenue will be $y_i = p_i x_i(p_i)$ and

$$\delta y_i/\delta p_i = x_i(1 + p_i x_i'/x_i).$$

It follows that (p_i) = (p^c) is a Nash equilibrium *only if* every contingent individual demand curve is elastic ($-p_i x_i'/x_i \geq 1$) at p^c. Excluding exceptional cases (involving nonlinear demand curves) this is also a sufficient condition for Nash equilibrium.

As emphasized by Shubik (1959 ch. 5) the derivation of contingent demand curves is, in general, a complex problem, since it involves demand when more than one price exists for the same good. But, following Shapley and Shubik (1969), suppose, as an illustrative example, that demand conditions can be represented by an aggregate consumer with a quadratic utility function maximizing[5]

$$ax - (b/2)x^2 - \Sigma_j p_j x_j,$$

where x_j denotes purchases from firm j at price p_j and $x = \Sigma_j x_j$.

Putting $p_j = p$ we note that this behaviour generates the linear aggregate demand curve

$$a - bx = p.$$

With $p_j = p^c$, $j \neq i$, and $x = Q - q_i + x_i$ we then obtain $x_i = x_i(p_i)$ from the first-order condition

$$a - b(Q - q_i + x_i) - p_i = 0.$$

Substituting $a - bQ = p^c$ we find

$$x_i = q_i - (p_i - p^c)/b.$$

It follows that the contingent demand elasticity at $p_i = p^c$ is

$$-p^c x_i'(p^c)/q_i = p^c/bq_i = \frac{\eta(p^c)}{q_i/Q}$$

where $\eta(p^c)$ denotes the elasticity of the *aggregate* demand curve at $p = p^c$. We conclude that in this example $(p_i) = (p^c)$ is a Nash equilibrium only if, and excluding exceptional cases also if,

$$q_i/Q \leqslant \eta(p^c) \text{ for every i.}$$

This condition is always fulfilled (including the duopoly case) if aggregate demand is elastic at the competitive price. And equilibrium will also always be competitive if the market is atomistic in the sense that the output share of the biggest firm is less than the demand elasticity at the competitive price.

We now relax the rather restrictive assumption above that sellers cannot change prices offered at the beginning of the market period. Thus we assume that every firm is free to change its price at any time. We also assume that every seller – as well as every buyer – can observe the prices of all firms. Contemplating a price change a seller must consequently reckon with responses evoked by such a change. In this setting a firm's *pricing strategy* σ_i consists of an *initial price* p_i and a *response function* f_i,

$$\sigma_i = (p_i, f_i).$$

Following Marschak and Selten (1978) we now assume that response times are negligible – so that transitory profits can be ignored – and introduce the "kinked" response function min p_j or in full

$$p_i = \max \{ p^c, \min \{ p_j \} \} .$$

Having observed prices p_j, firm i immediately conforms to the lowest bidder, according to this response function. (But of course it never goes below p^c.) Consider the strategies

$$\sigma_i(p) = (p, \min p_j),$$

implying that every firm quotes p initially and then behaves according to the kinked response function (matches every price cut instantaneously). To simplify the exposition we assume – recalling that $(pD)' = D(1 - \eta)$ – that pD(p) is increasing up to p^o defined (uniquely) by $\eta(p^o) = 1$, and then decreasing.

PROPOSITION. The pricing strategies $\sigma_1(p)$, $\sigma_2(p)$,...$\sigma_n(p)$ are in equilibrium for $p > p^c$ if aggregate demand is inelastic at p ($\eta(p) \leqslant 1$) and if aggregate excess supply at p is larger than every individual supply ($Q - D(p) > q_i$ for every i).

Note that the proposition applies to every n, including "perfect competition" ($n \to \infty$). To prove it we consider seller i and let him take his rivals' *complete* pricing strategies $\sigma_j(p)$ as given – not merely their initial prices $p_j = p$ – when contemplating defections $p_i \neq p$ from the initial price p. For $p_i < p$ (price cutting) the market price will of course not be p but p_i, due to the rivals' kinked response functions. Then individual sales will be $\alpha_i D(p_i)$, where we can take the market shares α_i to be exogenously

given (excluding advertising and assuming outputs q_i to be exogenously given). It is then obvious that $p_i = p$ will maximize individual revenue $\alpha_i p_i D(p_i)$ subject to $p_i \leq p$ if (and only if) $\eta(p) \leq 1$. If moreover

$$D(p) < Q - q_i$$

every consumer can turn to rivals selling at p if i quotes $p_i > p$, making a price rise unattractive. Note that the condition above will always be fulfilled for $p > p^c$ if individual supplies are sufficiently small.

Excluding exceptional cases involving big firms, which might make price increases appear profitable, we conclude that there exists a symmetric ($p_i = p$ for every i) excess supply equilibrium if aggregate demand is inelastic at the competitive price. Equilibrium is not unique but this is no problem here. In fact every seller will prefer p^o defined by $\eta(p^o) = 1$ since p^o maximizes every seller's individual revenue $\alpha_i pD(p)$. Hence the point associated with p^o is simultaneously a noncooperative equilibrium and a profit frontier point and thus a perfect candidate for tacit collusion (cf. J. Friedman 1983 p. 132).

When aggregate demand is inelastic at the market-clearing price every seller prefers that price which maximizes collective sales revenue pD(p). It is true that this price (p^o) is not supported by noncooperative behaviour when price-setting is decentralized to sellers *if price-setters have to stick to their initial price quotations throughout the market period.* But if firms are free to revise their prices at any time, p^o will indeed be supported by noncooperative behaviour. For then no seller has anything to loose by quoting p^o initially, since he can always match a price cut. Moreover, there is no incitement to initiate a price cut – when demand is inelastic – since rivals will always retaliate.

These ideas are certainly not new. Our point is that the literature on industrial organization (see Scherer 1980 for example) does not have to apologize for the fundamental insight that price competition is prevented by threats to match price cuts. While competitive pricing presupposes prices that are rigid during the market period, the formal justification of noncompetitive pricing (when demand is inelastic) relies on perfectly flexible prices. In this case pricing in effect becomes a *repeated game* or an iterated Prisoner's Dilemma. The evolution of tacit collusion in such situations has been emphasized not least by J. Friedman (1983) and Axelrod (1984).

4. *Rational pricing when supply is perfectly elastic*

To see that noncompetitive pricing is not a fixed supply artefact we shall now discuss an example with production. For simplicity we assume that average and marginal costs are equal and constant (c) and we take the

number of firms to be exogenously given (n), excluding the possibility of entry. If capacities (k_j) are limited, so that a firm's rivals cannot satisfy the whole market ($\Sigma_{j\neq i} k_j < D(c)$), a firm's sales do not reduce to zero for $p_i > c$, given $p_j = c$ ($j \neq i$). In this case the market price is *indeterminate*, as emphasized by Edgeworth (1925 p. 120), Shubik (1959 p. 92) and Shapley and Shubik (1969). A Cournot-Nash equilibrium does not exist. In the following analysis we consequently exclude this case. On the other hand we do *not* exclude the case when every firm has enough capacity to satisfy the whole market ($k_j \geq D(c)$).

We assume that consumers place orders with the lowest bidder(s) and that firms produce what has been ordered from them. Only with this arrangement can output be prevented from being a producer's decision variable. It is determined by consumers. Market-clearing is an automatic consequence of this set-up.

The customary presumption in price theory is that firms producing perfectly homogeneous goods cannot select their own prices without being forced to marginal cost pricing – even when there are only two firms. The well-known text-book argument (following Bertrand) is that $p_i = p > c$ cannot be an equilibrium point since every firm can increase its sales discontinuously by choosing a price slightly less than p. More precisely $p_i = p$ is not optimal against $p_j = p$ ($j \neq i$) for any $p > c$, while $p_i = c$ is optimal against $p_j = c$ ($j \neq i$) for every i, since by assumption i's rivals have sufficient capacity to satisfy the whole market.

It is not always appreciated, however, that the Bertrand story crucially depends on *sealed bidding* with prices that cannot be changed. Only in this case can a pricing strategy be represented by a single number. This is not possible in an industry where firms can observe each others' prices and are free to match price cuts at any time. For then a pricing strategy consists of 1) an initial price p_i and 2) a response function f_i, exactly as in section 3. Consider the strategies

$$\sigma_i(p) = (p, \min p_j),$$

implying that every firm quotes the same initial price and then behaves according to the kinked response function introduced in section 3. Let p^m maximize collective profits $(p - c)D(p)$. To simplify the exposition we assume that $(p - c)D(p)$ is increasing up to p^m and then decreasing. Consider $p \geq c$.

PROPOSITION. $\sigma(p) = (\sigma_1(p), \sigma_2(p), \ldots \sigma_n(p))$ is an equilibrium point for every $p \leq p^m$.

To see this we first observe that given a uniform price (p) no consumer will prefer to place his orders with any particular firm (since products are perfect substitutes). Hence we may safely assume that a firm's sales will be $D(p)/n$ and its profits

$$\pi_i(p) = (p - c)D(p)/n.$$

Taking σ_j, $j \neq i$, as given, $p_i < p$ (price cutting) will thus yield $(p_i - c)D(p_i)/n$ to producer i *when his rivals' reactions have been taken into account*. Hence price cutting will appear profitable to i *only if* $(p - c)D(p)$ increases when p decreases, i.e. only if $p > p^m$. Finally, since a firm's rivals can satisfy the whole market (by assumption) a price rise will never be attractive.

In fact it is apparent that every firm will prefer the monopoly price p^m. Realizing this, and realizing that this price is supported by noncooperative behaviour when firms are free to match price cuts, every firm will quote p^m initially – and stick to it throughout the market-period.

Summarizing our discussion in this section we have found on one hand that marginal cost pricing is inevitable in a market with sealed bidding and production to orders. This result is valid not only in the limit $(n \to \infty)$ but for any number of firms, including duopoly.[6] On the other hand we have found that monopoly pricing persists in a market where firms produce to orders if firms are free to change their prices at any time. This result is also valid for any number of firms, including the limiting case of pure competition $(n \to \infty)$.

5. *A vision of the price mechanism*

The most important moral of our examples is perhaps that the character of an equilibrium position crucially depends on the way the market is organized. In an auction market the equilibrium price depends upon the number of firms, and it approaches the Walrasian price when the number increases (section 2). On the other hand numbers do not matter when firms produce to orders (section 4). Instead the outcome depends on whether there is sealed bidding or not.

When firms produce to orders or sell their outputs in an auction market, market-clearing is guaranteed by the institutional setting. But in Marshall's corn-market example, discussed in section 3, stocks remaining at the end of the market period are determined endogenously. If there is sealed bidding, price-setting suppliers, taking their supplies and market shares as given, will choose the market-clearing price (when a Cournot-Nash equilibrium exists). But if they are free to change their prices at any time, the character of the equilibrium position depends upon the elasticity of the aggregate demand curve. The market will clear endogenously if demand is elastic. But confronted with an inelastic aggregate demand curve suppliers will choose that price which maximizes their collective sales revenues. Some stocks will remain unsold at the end of the market period since sellers do not find it profitable to lower their prices to get rid of them.

But what happens if supplies are no longer taken as given but produced before the market period by firms predicting unsold goods at noncompetitive prices? This problem of simultaneous price and quantity adjust-

ment is adressed in subsequent chapters. We find that as a rule (with some minor exceptions) independent firms quote that price (p^o) which maximizes the industry's collective sales revenues $pD(p)$ for prices above the market-clearing level, $p \geq p^c$. The character of the equilibrium position will consequently depend on the elasticity of the aggregate demand curve. Inelastic demand ($p^o > p^c$) implies a noncompetitive equilibrium price (p^o) for atomistic as well as oligopolistic industries. On one hand the oligopoly price will never approach the monopoly price as long as firms are free to produce what they like at the ruling market price. On the other hand price competition forcing the price down to p^c will never be rational for atomistic firms as long as they are free to change their prices at any time.

Realizing that they cannot sell everything supplied at a non-competitive price, producers will adjust their outputs accordingly. The rate of excess supply in (Cournot) equilibrium at p^o will be less than $1 - D(p^o)/S(p^o)$. In atomistic markets it is $1 - d(p^o)$, where $d = d(p)$ is defined implicitly by the equation

$$D(p) = S(pd)d.$$

In an oligopolistic market with a horizontal industry supply curve and n firms it is equal to $(p^o - p^u)/p^o$, where $p^u = p^c/(1 - 1/n)$.

The analysis of atomistic product markets in chapter 4 is extended to labour markets in chapters 5 and 6, assuming that workers have a purely instrumental attitude towards time spent in market activities, so that the division of this time between work and job search does not matter in itself, only the total payoff in terms of goods.

The customary labour supply curve aggregates individual supply curves derived on the assumption that employment will be equal to supply. But this assumption is not valid when there is unemployment. Rational workers should realize this and adjust their supplies accordingly. In chapter 5 the corresponding "effective" labour supply curve is derived and used to derive the true rate of unemployment and explain why a rise in the demand for labour may increase its supply (the added worker effect).

In chapter 6 the core of my message – the "collective-revenue-maximization-hypothesis" – is extended to the labour market. In this case it can be interpreted as a choice-theoretic rationale for the classical notion of wage bill maximization as the proper economic goal for a trade union. We also find that the wage-bill-maximizing wage is supported by non-cooperative behaviour when (reservation) wages are perfectly flexible.

Following Chamberlin (1962) and Diamond (1971) the possibility of noncompetitive equilibria in atomistic industries has been demonstrated for markets with *imperfect competition*[7] or *imperfect information*.[8] In this literature individual firms are given a certain amount of market power – summarized in an imperfectly elastic individual demand curve – through the introduction of an "imperfection". This imperfection is either *differentiated products* (in the literature on monopolistic competition) or

consumers' incomplete information on firms' prices (with the search theoretic approach).

But noncompetitive results in models with a noncompetitive structure ("imperfection") should come as no surprise. And how much do imperfections like differentiated products or incomplete price information matter? Equilibria may be qualitatively noncompetitive but quantitatively "almost" competitive – unless heterogeneity or uncertainty is a dominant or at least a prominent feature of the market structure. In the literature on monopolistic competition and incomplete information homogeneous markets and perfect information are also emphatically denied, the implicit assumption being, I suppose, that in a classical world prices would indeed be competitive.

It is usually taken for granted in economics that excess supply equilibria in markets with a competitive structure must be due to price rigidity.[9] The most paradoxical proposition of the present study is perhaps that it is *competitive* pricing which is due to price rigidity, while perfectly flexible prices entail noncompetitive pricing (when market demand is inelastic). To resolve this paradox we first observe that the usual dismissal of the possibility of noncompetitive states being noncooperative equilibria crucially depends on the assumption that a firm's pricing strategy can be represented by a single number – and this is only possible when a firm is committed to its initial (list) price until the beginning of the next market period. When firms are free to change their prices at any time during the market period such simple representations of pricing strategies are no longer possible. And in the resultant *infinite*-dimensional non-zero-sum game noncompetitive equilibria turn out to be possible.

The basic economic idea is simple enough: when prices are perfectly flexible a firm has nothing to loose by quoting a "high" price (when demand is inelastic) since it can always match a price cut. Hence rational producers should not hesitate to begin the market game with "high" list prices. And threats to match price cuts will then prevent price cuts (assuming perfect information on prices).

Formally, however, we are dealing with the tricky subject of infinite games. A completely rigorous analysis of pricing as an infinite sequential game is outside the scope of the present study. But a reappraisal of the consequences of perfect price flexibility is important enough to warrant at least one attempt at an axiomatic treatment. This is done in the next chapter, with Jörgen Weibull, in the simplest possible framework, viz. Marshall's corn market example.

Chapter 2

Noncooperative flexible pricing in a homogeneous market*

Ante Farm and Jörgen W. Weibull

1. Introduction

Economic theorists have in recent years devoted much attention to the foundations of the neoclassical doctrine of perfect competition. The motive and scope of this strand of research was well expressed by Andreu Mas-Colell in a special issue of the Journal of Economic Theory devoted to noncooperative approaches to this topic:

> "Modern Walrasian economics ... provides an analysis of the decentralized economic coordination problem under the hypothesis that prices are publicly quoted and are viewed by economic agents as exogenously given. If we regard this as the Hypothesis of Perfect Competition, then modern Walrasian economics is a theory of perfect competition only in the sense of examining the consequences of this Hypothesis but not in that of giving a theoretical explanation of the Hypothesis itself." (Mas-Colell (1980, p.121))

Also the present note takes a noncooperative approach, but differs from other studies in that it is focused on the classical case of homogeneity, perfect information, and, in particular, the limiting case of perfectly "flexible" prices in which each seller is free to instantly and costlessly revise his price when observing the prices of his competitors. To discuss these basic principles of decentralized price setting in the simplest possible framework, we turn to Marshall's corn market example, in which stocks offered for sale are in existence already at the outset, and where equilibrium is established through pure price adjustment (Marshall (1920, book V, p.278)).[1]

In this simple setting, sufficient conditions are given under which noncooperative equilibrium prices exist. Let p^c and p^o denote the competitive (Walrasian) price, and the price that maximizes the value of aggregate demand, respectively. If $p^o \leq p^c$ ("elastic demand"), then p^c is the only possible equilibrium price, while in the opposite case ("inelastic demand") p^o turns out to be an equilibrium price. In fact, it is then the unique equilibrium price on the Pareto frontier, and one can show that all individual strategies supporting p^o are equivalent – hence this price is an

ideal candidate for tacit collusion (cf. Friedman (1983, p.132)).[2] The result is shown to be valid for any fixed number n of sellers, as well as in the limit when n tends to infinity.

2. *Preliminaries*

Let $q_i > 0$ denote the (exogenous) supply of seller i, and let $p_i \geq 0$ be his price. Let $Q = \Sigma q_i$, and let the aggregate demand function D be continuous and strictly decreasing. Suppose $D(p^c) = Q$ for some $p^c > 0$ – then p^c is the (unique) *competitive price*. We assume that the aggregate expenditure function E, defined by $E(p) = pD(p)$, has a unique maximum at a positive price p^o, the "optimal" price, to the left (right) of which it is strictly increasing (decreasing).

If all sellers quote prices above p^c, then some sellers are rationed. In particular, if they all quote the same price $p > p^c$, then each seller's sales will be some share $\alpha_i(p)$ of his supply q_i. We assume $\alpha_i = \alpha_j$ for every i and j, i.e. every unit of the good has an equal "chance" of being sold.[3] In this sense the good is absolutely homogeneous. It follows that, at any uniform price p, $\alpha(p)Q = D(p)$, so individual trades are then given by

$$t_i = \min \{ q_i, q_i D(p)/Q \} \qquad (i = 1,\ldots,n). \qquad (1)$$

We assume that unsold goods are worthless to the sellers, i.e. they simply seek to maximize their sales revenues, $p_i t_i$. (This assumption can be somewhat relaxed, see discussion in Section 4 below.)

In the traditional game-theoretic underpinning of the Walrasian model, a seller's strategy is simply his price, and then it is evident that a uniform price above the competitive price cannot be a Nash equilibrium (NE). For if $p_j = p > p^c$ for all $j \neq i$, then a slight price cut, $p_i < p$, will yield a discontinuous increase in the revenues of seller i.[4] Note, however, that this conclusion relies on an implicit assumption of price rigidity (or commitment). For while the buyers are assumed to observe all prices before making purchases, the sellers are not permitted to change prices once announced. If instead each seller were free to revise his price after having observed his competitors' prices, then a seller wouldn't rationally expect competitors not to react to his deviation – the "no-response" hypothesis above would then at best be a special case to be deduced from anticipated rational behaviour on behalf of the competitors.

Of course, competitors (as well as buyers) react only with some delay and at certain costs in practice. However, in order to highlight the role of price flexibility for the character of equilibrium, we focus on the limiting case of instantaneous and costless price revisions before trade commences.[5]

3. *The model*

More precisely, we consider a tatonnement process, without an auctioneer, in which each seller observes all prices and sets his own price. At the outset, all sellers simultaneously announce their initial prices, $p(0) = (p_1(0),..,p_n(0))$, without cooperation. Immediately afterwards, they observe this price vector and are free to instantly and costlessly revise their prices. Hence, a new price vector $p(1)$ immediately arises, where $p_i(1) = f_i(p(0))$ and each function f_i is selected by seller i himself. For simplicity, we now restrict his further price revisions to follow the scheme $p_i(t) = f_i(p(t-1))$ for all $t > 0$, where f_i is continuous. In other words, the sellers are allowed to react to each other's prices, but only in terms of a stationary response function. Traditional pricing is obtained if one further restricts response functions to fulfill the requirement $f_i(p) = p_i$, so the present extension seems to be the simplest possible representation of perfect price flexibility.[6]

If the generated price sequence $\{ p(t) \}$ converges, then its limit is the associated *trading-price vector* ϱ. If the sequence diverges, then it is not evident how trading-prices should be determined. For definiteness, we let the competitive price then be the uniform trading price:

$$\varrho = \begin{cases} \lim_{t \to \infty} p(t) & \text{if } \{ p(t) \} \text{ converges} \\ (p^c,p^c,..,p^c) & \text{otherwise.} \end{cases} \qquad (2)$$

Individual trades $t_i(\varrho)$ are supposed never to exceed the corresponding supplies, $0 \leq t_i \leq q_i$, and $t_i(\varrho)$ is assumed to satisfy eq.(1) whenever trading prices are identical at all sellers. Moreover, since the good is homogeneous, we suppose that the buyers turn in priority to the sellers with the lowest price, or more exactly, that $t_i(\varrho) = 0$ if $\varrho_j = \varrho_k < \varrho_i$ for all $j,k \neq i$ and $Q - q_i \geq D(\varrho_j)$. Individual revenues are obtained by simply multiplying trades with trading prices: $\pi_i(\sigma) = \varrho_i t_i(\varrho)$.

In sum: we have defined an infinite-dimensional non-zero-sum game on normal form, in which a (pure) strategy σ_i is a pair $(p_i(0),f_i)$, where $p_i(0) \in R_+$ and $f_i:R_+^n \to R_+$ is continuous. Combined strategies are written $\sigma = (\sigma_1,...,\sigma_n)$ and the combined response function f is defined by $f(p) = (f_1(p),...,f_n(p))$. Individual payoffs associated with any combined strategy σ are $(\pi_1(\sigma),.., \pi_n(\sigma))$.

By continuity, only fixed points of f, i.e. price vectors at which no seller wants to change his price (under the restrictions of the game), can be limit points of the above tatonnement process. In particular, no price revision actually takes place if the initial price vector $p(0)$ is a fixed point of f – then $p(0)$ is itself the trading-price vector. This obervation suggests the following definition of equilibrium: a strategy vector $\sigma = ((p_1,f_1),..,(p_n,f_n))$ is an *equilibrium* if $p = f(p)$ and

$$\pi_i(\sigma_1,..,\sigma_i,..,\sigma_n) \geq \pi_i(\sigma_1,..,\sigma_i',..,\sigma_n) \qquad (3)$$

for every seller i and strategy $\sigma_i' = (p_i',f_i')$. In other words, an equilibrium is a Nash equilibrium in strategies with the further property that initially quoted prices are also trading prices.

Of particular relevance for economic modelling are *single-price equilibria* (SPE), i.e. equilibria σ with $p_i = p_j$ for all i and j. The subsequent analysis will be confined to this type of equilibrium. Let P be the corresponding (possibly empty) set of equilibrium prices, i.e. $P = \{ p' \in R_+ : ((p',f_1),..,(p',f_n))$ is a SPE for some $(f_1,..,f_n) \}$.

It follows immediately from our definition of equilibrium that the "no response" strategy ($f_i(p) = p_i$) does *not* support the "optimal" price p^o as a SPE. For any seller could then obtain a discontinuous increase in revenues by deviating from this strategy by a slight price cut in the "inelastic" case $p^c < p^o$, or a slight price rise in the opposite case. However, the price p^o will indeed be supported by certain other (noncooperative) strategies in many situations.

This result will be shown to hold under a simple but powerful condition. Let p^e be the *effective* optimal price, defined as the "optimal" price p^o multiplied by the corresponding rationing factor, i.e. $p^e = p^o\alpha(p^o) = E(p^o)/Q$. It follows from our assumptions that $p^c < p^e < p^o$ in the "inelastic" case.[7] The condition requires sellers to be sufficiently small in the sense that all individual supplies should be smaller than aggregate excess supply at p^e:

$$C : \max \{ q_i \} < Q - D(p^e).$$

Proposition: If $p^o > p^c$ and C holds, then $p^o \in P$.

Our proof of this result (see below) is based on a modification of the *kinked response function* (cf. Sweezy (1939) and Marschak and Selten (1978)). More exactly, in the "inelastic" case $p^c < p^o$, one can show that any strategy vector σ^o, with identical components (p^o,f^o), where

$$f^o(p) = \max \{ p^e, \min \{ p_1,..,p_n \} \} \text{ for all } p, \qquad (4)$$

indeed is a SPE.[8] For then no seller has an incentive to initiate a price cut, since competitors will retaliate (otherwise profitable) price cuts, and hence make every price cut unprofitable. The above proposition may therefore be viewed as a formalization and slight extension of the insight in the literature on industrial organization (see e.g. Scherer (1980, ch.5)) that price competition may be prevented by threats to match price cuts, an idea originally due to Chamberlin (1962, ch. 3).[9]

Proof of the proposition: Let $i \in \{ 1,..,n \}$ be arbitrary, and let $\sigma_j = (p^o,f^o)$ for all $j \neq i$. For any strategy σ_i, let σ be the corresponding strategy vector and ϱ the trading-price vector. If $\sigma_i = (p^o,f^o)$, then clearly $\varrho_i = p^o$ and $\pi_i = q_iE(p^o)/Q = p^eq_i$. If σ_i is such that $\{ p(t) \}$ diverges, then $\pi_i = p^cq_i < p^eq_i$. In case of convergence, the necessary condition $\varrho = f(\varrho)$

implies $\varrho_j = \varrho_k = f^o(\varrho)$ for all $j,k \neq i$. If $\varrho_j = \varrho_i$, then, by eq.(1), $\pi_i = \min \{ \varrho_i q_i, q_i E(\varrho_i)/Q \} \leq q_i E(p^o)/Q = p^e q_i$. If $\varrho_i < p^e$, then clearly $\pi_i < p^e q_i$. It hence remains to check strategies σ_i such that $\varrho_i \neq \varrho_j$ and $\varrho_i \geq p^e$. Then $p^e \leq \varrho_j < \varrho_i$ since $\varrho_j = f^o(\varrho)$ for all $j \neq i$, so $\pi_i = 0$ by C. The index i being arbitrary, it follows that σ^o satisfies (3). *End of proof.*

In fact, the difficulty here is not the existence of equilibria but their multiplicity. However, under the hypothesis of the proposition, the price p^o is actually the unique (uniform) equilibrium price that is not Pareto dominated, and all strategies supporting p^o are equivalent (in the sense of Luce and Raiffa (1957, p.106)). Hence p^o is an ideal candidate for tacit collusion (cf. Friedman (1983, p. 132)). The point is that no seller has anything to loose by quoting p^o initially, since he can always match a price cut.

More exactly, suppose $p^c < p^o$, and let P^* be the subset of *Pareto efficient prices* in P, i.e. (uniform) prices supported by SPE strategy vectors that are not dominated, in terms of individual revenues, by other SPE vectors. Assuming, as we do, that p^o is the unique maximum of the aggregate expenditure function E, it follows directly that $P^* = \{ p^o \}$, i.e. the price p^o is indeed optimal. Moreover, if $\sigma^1,..,\sigma^n$ are SPE strategy vectors supporting p^o, then any mixed strategy vector, e.g. $(\sigma_1^1,\sigma_2^2,..,\sigma_n^n)$, can easily be shown to also support p^o.[10] In other words: all Pareto efficient (single price) equilibria give the same maximal payoffs to all players, irrespective of which particular individual strategies in this set have been selected. Hence, individual rationality alone, in the sense of Johansen (1982) leads the sellers to choose (any) strategies supporting the optimal price p^o.[11]

This result is clearly valid for any fixed number n of competitors and any supply vector $q = (q_1,..,q_n)$ satisfying condition C. In particular, if each seller becomes less and less significant in the sense that n increases while aggregate supply Q is constant and no individual supply increases, then condition C just becomes more and more easily satisfied. Furthermore, it is readily verified that p^o remains an equilibrium price even when aggregate supply increases along with the number of competitors. For let the aggregate demand function D be fixed and given, and let $q = (q_1,\ldots,q_n)$ be a fixed supply vector. For any positive integer m, let mq be the m-fold replication of q. Since D is fixed, so is p^o, while for m sufficiently large, the corresponding competitive price $p^c(m)$ (defined by $D(p) = mQ$, granted $D(0) = +\infty$ and $D(+\infty) = 0$), falls below p^o. In other words, if we duplicate the supply side sufficiently many times, we sooner or later wind up in the "inelastic" case $p^c < p^o$. Moreover, the largest individual supply, $\max \{ q_i \}$, does not increase in this process, while the corresponding effective price, $p^e(m) = E(p^o)/(mQ)$, decreases monotonically towards zero – a natural effect of stiffening competition. Nevertheless, excess supply at this price rises monotonically. For

$$mQ - D(p^e(m)) = (1 - E(p^e(m))/E(p^o))mQ, \qquad (5)$$

where the right-hand side, for $p^e(m) < p^o$, is increasing in m (in fact toward infinity), since by assumption E(p) is increasing for $0 < p < p^o$. Hence condition C holds for all m above a certain level, and p^o remains an equilibrium price, also in the limit as the number of competitors increases towards infinity.

4. *Extensions*

Conspicuous restrictions in the present model are the assumptions (i) that supply is exogenously fixed, (ii) that pricing strategies are stationary, and (iii) that unsold goods are worthless to the seller. As for the first two aspects, extensions will be presented in subsequent studies (see Chapters 3 and 4 below and Weibull (1986)). Concerning the third restriction, on the other hand, a few words will be said here.

If the good is perishable, then assumption (iii) is natural, while if the good is storeable, then one can argue that the present model does not allow the agents to fully exploit potential gains of trade. However, suppose we extend the model by assigning an exogenously fixed and non-negative value r to each unit of unsold goods ("the expected present value of net revenues from future sales"). Then individual profits are given by the equation $\pi_i = \varrho_i t_i + (q_i - t_i)r = (\varrho_i - r)t_i + q_i r$. The last term, $q_i r$, is exogenously fixed, so maximization of $(\varrho_i - r)t_i$ is the relevant objective for seller i. Let $p^o(r) = \min \arg\max (p - r)D(p)$. (Note that $p^o(r)$ is formally identical with the monopoly price for an industry in which every firm produces according to the linear cost function $c_i(q_i) = rq_i$.) Then $p^o(r) > p^o$ for every r in $(0,p^o)$ and $p^o(0) = p^o$. Moreover, it is readily verified that if we, in the original model, replace p^o and p^e by the more general $p^o(r)$ and $p^e(r) = E(p^o(r))/Q$, then the proof of our proposition remains valid for any r in the interval $(0,p^o)$. Hence, if the model is closed with respect to trading possibilities in this fashion, then non-cooperative pricing in fact leads to an even higher equilibrium price.

Chapter 3

Oligopolistic pricing*

1. Introduction

In the literature on industrial organization it has been suggested that oligopolists are "likely to adopt a passive output determination policy, adjusting production to demand at the accepted price" (Scherer 1980 p. 193). One purpose of the present chapter is to provide a choice-theoretic rationale for this idea (in section 3) when the number of firms (n) is exogenously given (excluding the possibility of entry).

For simplicity we assume that average and marginal costs are equal and constant (c). In this simple framework we shall also analyse *cooperative pricing*, when oligopolists are able to make binding agreements on prices – but not on production. Let p^m denote the monopoly price, while p^o maximizes collective sales revenue $pD(p)$ for $p \geq c$. (We assume that p^m and p^o are unique and finite.) Excluding exceptional cases (noted in section 4) we shall see that cooperative pricing will be monopolistic if *and only if*

$$p^u \geq p^o \text{ and } n \leq \eta(p^m) \tag{1}$$

where

$$p^u = \frac{c}{1 - 1/n} \tag{2}$$

and η denotes the elasticity of the industry's demand curve, $\eta(p) = -pD'(p)/D(p)$. Otherwise the cooperative price will be $\max(p^u, p^o)$.

In the second half of the chapter we extend the analysis to *noncooperative pricing*. We find that p^o will be not only a profit frontier point but also supported by noncooperative behaviour if

$$p^o > \frac{p^u}{1 - 1/n}. \tag{3}$$

This condition reduces to

$$p^o > c, \tag{4}$$

when the market is "atomistic" ($n = \infty$).

Following the seminal work by Shubik (1959) we concentrate on homogeneous markets and the traditional variables in economics, price and quantity. We obtain our results on noncooperative market behaviour by extending oligopoly theory to include simultaneous price and quantity adjustment in markets where production precedes sales – as is the case in most product markets.[1] We also assume that firms are free to change

their (publicly announced) prices at any time. Then pricing in effect becomes a *repeated game*, an important generator of noncompetitive results, as emphasized not least by J. Friedman (1983). The original idea – the noncompetitive consequences of threats to match price cuts – is due to Chamberlin (1962 ch. 3) and since then frequently used (informally) in the literature on industrial organization (see e.g. Scherer 1980 ch. 5). We find not only that this idea can be applied to a homogeneous market but also that the formal development is, in fact, greatly facilitated by the classical assumption of homogeneity and the concomitant law of one price.

Another key element of the present approach is an explicit model of rationing and quantity adjustment in excess supply situations. Shubik (1959 p. 362) recognized the necessity of specifying a "convention to decide how the market is shared when the players' production oversaturates the demand at the price they both wish to charge" and also suggested *proportional rationing* (p. 106). In my view proportional rationing is not only an example of a "convention" but a rationing scheme that deserves special attention because it can be derived from more basic postulates about rationing through a matching process in an unregulated market. This is elaborated in the following section.

2. *Proportional rationing*

Let firm i produce and "bring to the market place" q_i units of a homogeneous good. Consider a uniform price ($p_i = p$). When aggregate supply ($Q = \Sigma_i q_i$) exceeds aggregate demand ($D = D(p)$) there will be *rationing*. To characterize rationing by the invisible hand when the good is homogeneous and the price uniform we can always write

$$r_i = \alpha_i D, \tag{5}$$

where r_i denotes individual sales and $\Sigma_i \alpha_i = 1$ if the market is frictionless, which we assume for simplicity. The market share α_i may or may not depend on q_i. If it doesn't, it is obvious that the profit-maximizing output will be $q_i = \alpha_i D$ and thus $Q = \Sigma_i q_i = D$, excluding excess supply in equilibrium. Moreover, in this case

$$\pi_i = \alpha_i(p - c)D(p) \tag{6}$$

suggesting monopolistic pricing when firms are free to match price cuts, exactly as when firms produce to orders (cf. chapter 1).

We now turn to the more interesting case when a firm can influence its sales r_i through its supply q_i and first observe that the outcome of an *unregulated matching process* involving interactions between buyers and sellers simply cannot, in general, be predicted with certainty. Thus rationing by the invisible hand is essentially *stochastic* and r_i a stochastic variable. But since we always have

$$r_i \leq q_i, \tag{7}$$

we also have

$$Er_i \leq q_i, \tag{8}$$

where Er_i denotes the expected value of r_i. Hence we can always write

$$Er_i = d_i q_i \tag{9}$$

where $d_i \leq 1$. We interpret d_i as the firm's degree of rationing and postulate that every firm is rationed to the same degree:

ASSUMPTION. (Anonymity). $Er_i/q_i = Er_j/q_j$ for every i and j.

$Er_i/q_i < Er_j/q_j$ would imply that a unit of supply from j had a higher probability of being sold than a unit from i, in spite of postulated product homogeneity and price uniformity. Excluding such biases amounts to excluding (unequal) advertising and discriminatory marketing technologies[2] from our (pure) model of the market mechanism.

Substituting $Er_i = dq_i$ in $E\Sigma_i r_i = D$ it follows that $d = D/Q$ and

$$Er_i = dq_i = (q_i/Q)D. \tag{10}$$

This is proportional rationing applied to expected values. To be able to conclude that

$$r_i = (q_i/Q)D \tag{11}$$

we have to add the assumption that

$$r_i = Er_i \text{ (with probability 1).} \tag{12}$$

When yearly sales (r_i) may be represented as a sum of more or less independent monthly or weekly sales this assumption may be rationalized by the (strong) law of large numbers, as in most markets for consumer goods.

3. *Effective supply in a fixprice economy*

In this section we assume that the market price p is exogenously given. When p is greater than c firms would produce at full capacity ($q_i = k_i$) if they believed that they would be able to sell everything supplied. But such beliefs would be verified by experience only if $\Sigma_i k_i = K \leq D(p)$. If

$$K > D(c) \tag{13}$$

firms cannot supply k_i in expectational equilibrium.

To derive *effective supplies* for $p > c$ when (13) applies we postulate *proportional rationing* and consider the payoff functions

$$\pi_i = pD(p)q_i/\Sigma_j q_j - cq_i, \tag{14}$$

defining a noncooperative game in quantities. Note that (14) is based on

the simplifying assumption that goods are perishable so that evaluation of stocks remaining at the end of the market period can be ignored.

Put $\Sigma_j q_j = Q$ and introduce the market shares

$$\alpha_i = q_i/Q, \; i = 1,2,...n, \tag{15}$$

and the degree of utilization or "employment"

$$d = d(p) = D(p)/Q. \tag{16}$$

Then we have

$$\delta\pi_i/\delta q_i = p(1 - \alpha_i)d - c \tag{17}$$

and we first note that $q = (q_i)$ will be an interior Cournot-Nash equilibrium if, for every i,

$$p(1 - \alpha_i)d = c. \tag{18}$$

Secondly we observe that a point on the demand curve (where $d = 1$) will be an equilibrium point if $\delta\pi_i/\delta q_i \leqslant 0$ for $\Delta q_i > 0$, i.e. if, for every i,

$$p(1 - \alpha_i) \leqslant c. \tag{19}$$

Thirdly we find that (17) still applies for $\Delta q_i < 0$ when firms produce at full capacity (when $\alpha_i = k_i/K$ and $d = D(p)/K$) so that a point on K (cf. figure 1) is an equilibrium point if, for every i,

$$p(1 - k_i/K)D(p)/K \geqslant c. \tag{20}$$

The solution to (18) is obviously

$$\alpha_i^e = 1/n \text{ and } d^e = \frac{c/p}{1 - 1/n}, \tag{21}$$

and this is indeed an interior solution if

$$D(p)/K < d^e < 1, \tag{22}$$

or, equivalently, if

$$p > \frac{c}{1 - 1/n} \tag{23}$$

and

$$\frac{pD(p)}{cD(c)} < \frac{K}{D(c)} \quad \frac{1}{1 - 1/n} \tag{24}$$

Condition (24) may be violated if total capacity is close to aggregate demand or aggregate demand very inelastic ($pD(p) >> cD(c)$). Excluding this case, however, the Cournot-Nash equilibrium is given by (21) if $p > c/(1 - 1/n)$.

If, on the other hand, $p \leqslant c/(1 - 1/n)$,

$$\alpha_i = 1 - c/p + \varepsilon_i \tag{25}$$

will satisfy (19) and the definitional requirement $\Sigma_i \alpha_i = 1$ provided that

$$\varepsilon_i \geq 0 \text{ and } \Sigma_i \varepsilon_i / n = 1/n - (1 - c/p). \tag{26}$$

Note that such ε_i exist when $p \leq c/(1 - 1/n)$. In this case – but only in this case – will endogenous output adjustment imply a market-clearing state.

Since

$$\pi_i = \alpha_i(p - c)D(p) \tag{27}$$

when $d = 1$ one might think that there would be a constant fight over market shares in a market-clearing state when $p > c$. Increasing output q_i also appears profitable to producer i as long as $\alpha_i < 1 - c/p$, but as soon as his market share exceeds $1 - c/p$ he does not, in fact, find it profitable to increase it any further, *taking other firms' output as given*. Being satisfied with a market share at least as great as $1 - c/p$ might be interpreted as "satisficing" behaviour. If every producer is a satisficer the race is over whenever a market-clearing state is reached and it so happens that $q_i \geq (1 - c/p)D(p)$ for every i. In this case $(p - c)(q_i - (1 - c/p)D(p))$ might be interpreted as "windfall profits".

But a producer could also be a "maximalist", attempting to reach the maximal market share compatible with Nash equilibrium on the assumption that others will be satisficers. Given an initial market-clearing state he would then deliberately increase his production, in spite of the fact that his profits will be reduced if other producers do not change their output levels. But in this venture the maximalist, of course, does not take other producers' output as *given*. Instead his purpose is to lure his competitors into *reducing* their output by creating excess supply and thus a situation where output reduction will appear profitable to them.

This story is plausible if there is one big firm in the market and the rest are small. A small firm cannot hope to create a noticable rate of excess supply in the market all by himself, but a big firm can. In this case the market share in equilibrium will be $1 - c/p$ for small firms and $1 - (n - 1)(1 - c/p)$ for the big one.

With more than one "maximalist" in the market, however, the situation will again be indeterminate – as long as the Cournot-Nash equilibrium is not unique. But even if market-shares are not uniquely determined by (25) and (26) they "tend towards uniqueness" (in fact towards $\alpha_i = 1/n$) when $p \to p^u = c/(1 - 1/n)$. And if the market price exceeds p^u the fight over market shares will stop, according to (21).

Note that the concept of a market-clearing price is not unique in the present context, even if it "tends towards uniqueness" when the number of firms increases. We may refer to p^u as the *upper market-clearing price*.

As illustrated in figure 1 the aggregate effective supply curve Q^e coincides with the aggregate demand curve D between the lower market-clearing price (c) and the upper market-clearing price p^u. Otherwise – and excluding exceptional cases (characterized by (20)) – aggregate effective supply is defined by the equation $Q = D(p)/d^e$ or, substituting (21),

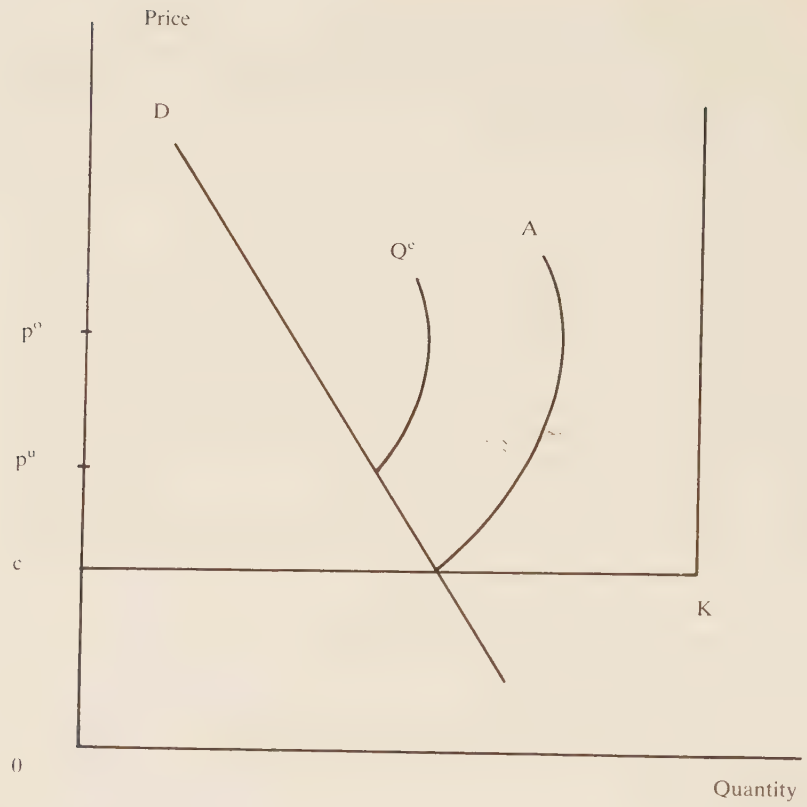

Figure 1. Effective supply (Q^e) of an oligopolistic industry and its atomistic limit (A).

Note: Demand is assumed (in the figure) to be inelastic at p^u. Q^e and A are perfectly inelastic at p^o, corresponding to unitary demand elasticity.

$$Q = (1 - 1/n)pD(p)/c,\ p \geq p^u. \tag{28}$$

Since $(pD)' = D(1 - \eta)$ this curve will be forward-bending at p if and only if aggregate demand is inelastic at p ($\eta(p) < 1$). When there are many firms the effective supply curve Q^e will be close to A – cf. figure 1. A represents effective supply in an "atomistic" market ($n \to \infty$). Thus its equation is

$$Q = pD(p)/c,\ p \geq c, \tag{29}$$

which happens to be identical to the equation for zero collective profits in the industry.

In the literature on industrial organization it has been suggested that oligopolists are "likely to adopt a passive output determination policy, adjusting production to demand at the accepted price" (Scherer 1980 p. 193). Our theory of effective supply provides a choice-theoretic rationale for this idea. Given a price above the competitive level there is nothing mysterious about endogenous output restriction by independent firms. Realizing that they cannot sell everything they supply, rational producers will adjust their outputs accordingly. But "adjusting production to demand" will be *complete* only if $p \leq p^u$ (cf. figure 1).

4. *Cooperative pricing without production quotas*

In this section we assume that oligopolists are able to make binding agreements on pricing – but not on production. When price quotations are freely observable – as in consumer markets – this might be a plausible assumption, provided, in addition, that agreements as well as sanctions can be kept secret (when illegal). In any case this is an interesting "intermediate case", between a fully cooperative approach to oligopoly theory, making the industry more or less equivalent to a monopoly in relation to its customers, and a fully noncooperative approach, to be developed in sections 5-9.

To derive the best price for pricefixing oligopolists we start from individual profits as a function of p after quantity adjustment according to section 3. In equilibrium at $p \geq c$ we have[3]

$$\pi_i = \pi_i(p) = \begin{cases} \alpha_i(p - c)D(p) & \text{if } p \leq p^u, \quad (30) \\ D(p)/n^2 & \text{if } p \geq p^u, \quad (31) \end{cases}$$

where p^u denotes the upper market-clearing price, $p^u = c/(1 - 1/n)$. (Note that $\pi_i(p^u) = cD(p^u)/n(n-1)$ according to (31) as well as (30), since $\alpha_i = 1/n$ for $p = p^u$.)

We have seen that $\alpha_i = 1/n$ in equilibrium for $p > p^u$. $\alpha_i = 1/n$ defines an equilibrium even if $p \leq p^u$ – according to (25) and (26) – but the equilibrium market shares are not unique in this case – unless $p = p^u$ – as emphasized in section 3. If, for example, there is one big firm in the industry and the rest are small, we have argued that the market share may be $1 - c/p$ for small firms and $1 - (n - 1)(1 - c/p)$ for the big one. Increasing p from c to p^u, the market share for small firms will *increase* from 0 to 1/n, while the market share for the big firm will *decrease* from 1 to 1/n. Here we shall ignore such complications and assume that the

market shares α_i are independent of p for $p \leq p^u$ and thus equal to 1/n, representing the case of *no dominant firm*. In that case every firm will prefer the same market price, namely that price which maximizes $\pi_i(p)$ according to (31) and (30) with $\alpha_i = 1/n$.

Let p^m denote the monopoly price, maximizing collective profits $(p - c)D(p)$, and p^o that price which maximizes collective sales revenue $pD(p)$ for $p \geq c$. If

$$(p - c)D(p) \text{ is an increasing function of p up to } p^m, \tag{32}$$

and

$$pD(p) \text{ is a decreasing function of p above } p^o, \tag{33}$$

the optimal cooperative price can be simply related to p^u, p^o and p^m. If $p^o > p^u$ – and thus[4] also $p^m > p^u$ – it follows from (31), (30) and (32) that the optimal cooperative price will be p^o. If on the other hand $p^o \leq p^u$ every producer will prefer

$$\min(p^u, p^m) \tag{34}$$

to be the market price, on account of (30), (31) and (33). It is easy to see[5] that producers will prefer the monopoly price p^m if and only if $n \leq \eta(p^m)$.

In other words cooperative pricing will be monopolistic if *and only if*

$$p^o \leq p^u \text{ and } n \leq \eta(p^m). \tag{35}$$

Otherwise the optimal price will be p^u if $p^o \leq p^u$ or p^o if $p^o > p^u$ or in short

$$\max(p^u, p^o). \tag{36}$$

This simple characterization of cooperative oligopoly pricing apply when (32) and (33) are valid – for example when the industry faces a linear demand curve – but note that these conditions are sufficient, not necessary. We may safely refer to cases where the general formula ($\max \pi_i(p)$ according to (30) and (31)) yields a divergent price as exceptional.

When there are many firms – in fact more than $\eta(p^m)$ – the cooperative oligopoly price is simply $\max(p^u, p^o)$. If $p^o = c$ this is equal to $p^u = c/(1 - 1/n)$ which approaches c with increasing n. If on the other hand demand is inelastic and $p^o > c$, $\max(p^u, p^o)$ will be equal to p^o for every n sufficiently large. With an increasing number of firms a noncompetitive price p^o will consequently persist, but note that total industry profit always approaches zero – according to (31) - as aggregate effective supply Q^e increases towards its atomistic limit A (cf. figure 1).

5. *Noncooperative price and quantity adjustment*

In section 4 oligopolists decide individually on outputs but collectively on the market price. We now exclude the possibility of binding agreements on pricing and ask ourselves if and when a noncompetitive price can be supported by *tacit* collusion – i.e. noncooperative behaviour – in industries where production precedes sales.

From now on firms consequenctly decide individually on prices as well as outputs. A price-quantity game was first formulated by Shubik (1959 p. 110). In his duopoly model the payoff function to player i is

$$P_i = p_i \min(q_i, D_i) - c_i(q_i) - I(q_i, D_i), \tag{37}$$

where (p_i, q_i) is the duopolist's price-quantity strategy (*two* numbers) and D_i his *contingent demand*, which is a function not only of the price charged by player i but also the price charged (p_j) and the quantity produced (q_j) by the other duopolist, $D_i = D_i(p_i; p_j, q_j)$. Note that the amount which player i sells will be q_i if $q_i \leq D_i$ and otherwise D_i. $I(q_i, D_i)$ is an *inventory evaluation function*, which, as Shubik emphasizes, in general must be included in the payoff function. Of course this component is zero if the producer sells everything, i.e. if $q_i \leq D_i$. It is *always* zero if we assume that *goods are perishable*, which we do for simplicity in what follows.

A solution to the price-quantity game starting from individual contingent demand functions has not been attempted in the oligopoly literature, with reference to the complexities involved in the derivation of the contingent demand functions.[6] A further complication in the traditional set-up is that a competitive state ($p_i = c$) can *never* be a noncooperative equilibrium, *not even when every firm has sufficient capacity to satisfy the whole market*.[7]

To prove this somewhat paradoxical result we begin by observing that in a price-quantity game not only other firms' prices but also other firms' output levels must be taken as given when proving Nash equilibrium. A firm's complete strategy is $\sigma_i = (p_i, q_i)$ and $(\sigma_1, \sigma_2, \ldots \sigma_n)$ – or (p,q) for short – is an equilibrium point only if σ_i is optimal when σ_j $(j \neq i)$ are taken as given.

A necessary condition for (c,q) to be a Nash equilibrium is that

$$\Sigma_i q_i = Q \leq D(c), \tag{38}$$

since otherwise

$$\Sigma_i \pi_i = c(D(c) - Q) < 0,$$

implying that $\pi_i < 0$ for at least one i (since $\pi_i \leq 0$). Now we take $\sigma_j = (c, q_j)$ as given for $j \neq i$ and contemplate strategies $(p_i, x_i) \neq (c, q_i)$ for firm i. Then consumers will buy $Q - q_i$ (but no more!) from the c-sellers at price c and the "contingent" demand x_i from firm i at price $p_i \geq c$. With

$$\pi_i = p_i x_i(p_i) - c x_i(p_i) \tag{39}$$

we always have

$$\delta\pi_i/\delta p_i = x_i(c) = q_i \text{ at } p_i = c, \tag{40}$$

so (c,q) cannot be a noncooperative equilibrium unless $q_i = 0$ for every i. We conclude that a competitive state which clears the market ($\Sigma_i q_i = D(c)$) can *never* be a noncooperative equilibrium in a price-quantity game in which the strategy of each player consists of two numbers, namely a price at which he will sell his product and a quantity which he will bring to the market place. An immediate corollary is that no (symmetric) equilibrium exists in this game. For taking $\sigma_j = (p,q_j)$, $j \neq i$, as given for $p > c$, a slight price decrease $p_i < p$ will always yield a discontinuous increase in individual sales and profits.

6. *Noncooperative price and quantity adjustment with flexible prices*

But the argument in the preceding section depends crucially on the implicit assumption that initial prices cannot be changed during the market period. Only in this case can a firm's pricing strategy be represented by a single number. Excluding this case, we now assume that prices can be revised (with negligible costs) at any time during the market period. We also assume that every producer – as well as every consumer – can observe the prices of all firms.

Contemplating a price change a firm must consequently reckon with responses evoked by such a change. In this setting a firm's *market strategy* σ_i consists of an *initial price* p_i and a *price response function* f_i in addition to its *output decision* q_i,

$$\sigma_i = (p_i,q_i,f_i). \tag{41}$$

Note that we continue to assume that a firm's output strategy can be represented by a single number. While prices can be revised at any time during the market period, output rates (production plans) cannot be revised until the beginning of the next market period, implying that "price is more flexible than quantity" in our model.

Following Marschak and Selten (1978) we now assume that response times are negligible – so that transitory profits can be ignored – and introduce the "kinked" response function

$$p_i = \max \{ c, \min \{ p_1,\ldots p_n \} \} \tag{42}$$

or min p_j for short. Having observed prices p_j, producer i immediately conforms to the lowest bidder, according to this response function. But he does not match "excessive" price cuts, below c.

Consider the market strategies

$$\sigma_i(p) = (p, q_i^e(p), \min p_j), \tag{43}$$

implying that every firm quotes p initially and produces the corresponding effective supply $q_i^e(p)$ and then behaves according to the kinked response function (matches every price cut instantaneously).

THEOREM. Consider an industry with n firms producing a homogeneous good in a market where production precedes sales and prices can be revised at any time. Average and marginal costs are equal and constant (c). Suppose that

$$p^o \geq \frac{p^u}{1 - 1/n}, \tag{44}$$

where p^o maximizes collective sales revenue $pD(p)$ for $p \geq c$ and p^u denotes the upper market-clearing price (introduced in section 3 and figure 1), $p^u = c/(1 - 1/n)$. Then $\sigma(p^o) = (\sigma_1(p^o), \sigma_2(p^o), \ldots \sigma_n(p^o))$ is a unique equilibrium point on the profit possibility frontier, implying that rational oligopolists 1) quote p^o initially and 2) stick to p^o throughout the market period. The corresponding rate of excess supply is $(p^o - p^u)/p^o$.

To prove this result we begin by taking the kinked response functions as given. Following Sweezy (1939) we thus simply assume that oligopolists are matching price cuts but not price rises. Unlike Sweezy, however, we shall not take current (i.e. initial) prices as given (by history), but attempt to explain current prices (and outputs) as rational choices in a noncooperative decision situation.

We begin by verifying that $\sigma(p)$ is an equilibrium point for $p = p^o$ under the given conditions. To do this we consider a producer i and let him take his rivals' strategies $\sigma_j(p)$ as given, with $p_j = p$ and $q_j = q_j^e(p)$. Let his initial price be p_i and his output q_i. If $p_i < p$ (price cutting) the trading price will not be p but p_i, due to the kinked response functions. Then his (contingent) profit function will be

$$\pi_i(p_i, q_i) = p_i D(p_i) q_i / \Sigma_j q_j - c q_i, \; p_i \leq p, \tag{45}$$

for every q_i so large that $D(p_i)/\Sigma_j q_j \leq 1$. (Smaller q_i cannot be profit-maximizing, since then $\pi_i = p_i q_i - c q_i$.) Next we note that no pair (p_i, q_i) such that $p_i < p$ can be profit-maximizing if

$$p_i D(p_i) < pD(p) \text{ for every } p_i < p \tag{46}$$

since then (p, q_i) is more profitable than (p_i, q_i), according to (45). But (46) is obviously true if $p = p^o$, so then price cutting cannot be attractive. Moreover, if

$$D(p) \leq Q^e(p) - q_i^e(p) \tag{47}$$

every consumer can turn to rivals selling at p if i quotes $p_i > p$. To see that this condition is equivalent to $p \geq p^u/(1 - 1/n)$ we divide by $Q^e(p)$ and then substitute α_i^e and d^e according to (21), recalling that $c/(1 - 1/n)$

$= p^u$. Next we observe that with $p_i = p^o$ it will be optimal to produce $q_i = q_i^e(p^o)$, since $q^e(p)$ is a Cournot-Nash equilibrium by construction. The rate of excess supply in equilibrium is obtained from (21).

So far we have taken the kinked response functions as given. Given excess supply in a homogeneous market with perfect information we also note that the strategy of matching price cuts – and avoid price rises – appears not only plausible but also necessary, since otherwise a firm risks losing all its sales. It remains to prove formally, however, that a kinked response function is indeed optimal against other kinked response functions. This will be attempted in the following section. Uniqueness and status as a profit frontier point will be discussed in section 8. An analysis of the case of elastic demand – excluded by (44) – concludes the chapter.

7. *On the rationality of kinked response functions*

To complete the proof of the market price theorem in section 6 we have to prove formally that kinked response functions are in equilibrium too and thus rational in the sense of Nash. To do this in a sequential setting we consider the market at time t (after the beginning of the market period) and assume that producer i as well as his rivals have kept their initial (list) prices unchanged (and equal to p) up to t. We wish to argue that an attempt to attract a discontinuous sales increase through a price cut will be neutralized by "instantaneous retaliation", implying that "transitory profits can be ignored". So far we have assumed this to be the case whenever "response times are negligible", but we now observe that "instantaneous retaliation" might in its turn be neutralized by "instantaneous trade", which we consequently have to exclude.

To see more clearly that our market price theorem relies on an important (but natural) implicit assumption about customer behaviour, we first imagine that every buyer (as well as every seller) is present at the market-place throughout the market-period. Then customers might refuse to buy until the market is about to close with a view to forcing sellers into the corner of a one-shot pricing problem. And even if this does not necessarily imply competitive pricing, as we saw in section 5, it entails indeterminacy.

In fact we face the same analytical problem in the beginning of the market-period. If trade could take place instantly in the sense that every consumer's purchases could be concentrated to the beginning of the market period, a firm's response function would not necessarily matter, only its initial price. Then a firm's pricing strategy could in effect be represented by a single number, excluding the possibility of equilibrium (according to section 5).

What we have to do to eliminate indeterminacy is to exclude the possibility of instantaneous trade. Suppose, to begin with, that customers

arrive stochastically, buy what they want (instantly) at the current price and then (immediately) leave the market place. To be specific we may assume that arrivals occur according to a Poisson process with parameter N, implying that the (expected) number of customers in a subperiod with length Δt is $N\Delta t$. Assuming in addition (for simplicity) that customers are identical, so that a consumer buys according to the demand function $D(p)/N$, total (expected) purchases in Δt will be $\Delta t D(p)$ when the (uniform) price is p in the subperiod $(t, t + \Delta t)$. This should be sufficient to motivate the exclusion of "instantaneous trade" in the following precise sense:

ASSUMPTION. Total demand at p in the subperiod $(t, t + \Delta t)$ is less than $h\Delta t D(p)$ for some constant h (independent of t and Δt).

We now return to the market at time $t > 0$ and assume that producer i as well as his competitors have kept their initial prices unchanged and equal to p up to t. Let i initiate a price cut $p_i < p$ at t. It follows immediately from our assumption on customer behaviour that the addition to i's profit in Δt due to a price cut $p_i < p$ will be less than $p_i h \Delta t D(p_i)$, and consequently also less than $hp^o D(p^o)\Delta t$.

Moreover, "instantaneous retaliation" according to the kinked response functions means that rivals will match the price cut within Δt for every $\Delta t > 0$, so that the trading price will certainly be uniform and equal to p_i after $t + \Delta t$. Suppose now that i's share of the market after $t + \Delta t$ is β_i without a price cut. Then a price cut at t can never increase this market-share, since with proportional rationing it is proportional to output remaining at $t + \Delta t$ and this output can never increase through a price cut at t. It follows that the change of i's profits on account of a price cut $p_i < p$ at t will be less than

$$hp^o D(p^o)\Delta t + \beta_i \gamma (p_i D(p_i) - pD(p))$$

for every $\Delta t > 0$, where γ denotes the fraction of the market remaining at $t + \Delta t$. (Note that neither γ nor β_i approaches 0 with Δt.)

We finally recognize that prices in practice cannot be changed continuously but only in multiples of one cent (say). Assuming that a price cut must have the form $p_i \leq p - \mu$, where μ is some small (but nonzero) amount, the profit change above will obviously be negative for some $\Delta t > 0$, provided of course that $p_i D(p_i) < pD(p)$ for every $p_i < p$. Then a price cut cannot be attractive.

In real life large price cuts may occur regularly at the end of the season (bargain sales). But this phenomenon must be interpreted as price discrimination – which is outside the scope of the present model – and not as a price war disproving the rationality (stability) of kinked response functions. For with perfectly flexible prices even at the end of the market-period a seller knows that he alone cannot get rid of individual supplies through price shaving. Only a (uniform) price cut which is sufficiently large to clear the whole market can do the job.

8. *A perfect candidate for tacit collusion*

In traditional single-period models of oligopoly behaviour, the Cournot-Nash equilibrium does not lie on the firms' profit possibility frontier, i.e. a simultaneous price increase would increase the profits of all firms. This uncomfortable fact has made many economists sceptical of Cournot-Nash equilibria in oligopoly theory.[8] On the other hand modern *multiperiod* models can generate noncooperative equilibria on the profit possibility frontier, as emphasized not least by James Friedman (e.g. 1983 pp. 123-134). A remaining problem in Friedman's models, however, is lack of uniqueness, suggesting that it would be difficult for firms to reach tacit collusion without communication.

Our theorem is akin to Friedman's results on self-enforcing agreements in multiperiod models in the fundamental sense that the equilibrium associated with p^o is 1) a noncooperative equilibrium and 2) a profit frontier point. The last fact follows immediately from section 4 (given the structural assumption that firms are always free to adjust their outputs to the ruling market-price). There are two important differences, however, which both can be characterized as a "return to the classics".

While Friedman starts by "replicating" the traditional single period we have started by "subdividing" it, or, to be more exact, by emphasizing the consequences of perfect price flexibility within the classical single-period model. Moreover, while Friedman follows the main stream of oligopoly theory since Hotelling (1929) and Chamberlin (1962) in starting from *differentiated* products, our point of departure is the classical notion of a market where firms produce *homogeneous* goods. There are two reasons for doing this, the basic reason being that the classical concept is a useful *first approximation* of real markets. Of course different firms never produce exactly the same products, but a producer does distinguish between rivals producing close substitutes and other firms. In my view, claiming that "homogeneous product markets are virtually nonexistent" (Friedman 1983 p. 75) is to throw the baby out with the bath-water. The concept of differentiated products is an important modification of traditional theory – which certainly adds to its realism – but it can not change its fundamental structure, as emphasized by Shubik (1959).

The second reason for preferring the classical homogeneous approach to the modern heterogeneous one is that it is analytically simpler, in spite of, or in fact rather because of, its dreaded discontinuities. Taking his rivals' prices as given, a producer can increase his sales discontinuously by undercutting their prices ever so slightly. Ever since Hotelling remarked, in his classical paper (1929), that "a discontinuity, like a vacuum, is abhorred by nature", discontinuities have been abhorred by most economists as well – perhaps for the reason that discontinuities have appeared to be impossible to handle with traditional tools. Making the model continuous – by introducing transportation costs or imperfect information – has been the "obvious" way out of this dilemma. But this

solves no problem, as emphasized by Shubik (1959 p. 98). Admittedly discontinuities of individual contingent demand functions are smoothed out, but nonconvexities and multiple maxima of individual demand functions are *not* eliminated. And it is irregularities of this kind which makes analyses involving individual demand functions so intractable, not discontinuities.

My conclusion has been to do away with individual demand functions, not discontinuities. In the present approach to oligopolistic pricing firms take the *aggregate* demand function as given, not *individual* demand functions (which are extremely hard to estimate objectively anyway; cf. Shubik 1959 p. 82). We keep the discontinuities to be able to exclude price differences in excess supply equilibrium, the intuitive argument being that even a slight price cut (rise) would entail such an increase (decrease) of a producer's market share that it could not be tolerated by his rivals (himself). More exactly we simply postulate that there are no (interesting) asymmetric equilibria in a homogeneous market with perfect information.

Is this a strong assumption? In modern price-setting literature, with its emphasis on individual demand curves, single-price equilibria appear to be just one out of many possibilities, and in fact often negligible or "of measure zero", as Shubik once put it (1959 p. 362). Once again I think this is to throw out the classical baby with the bath-water. Asymmetric equilibria may exist in homogeneous markets, but since examples of such equilibria are so hard to construct – *assuming perfect information and negligible transportation costs* – I believe that they can be treated as *exceptions* to the classical law of one price.

Excluding exceptional cases it follows that the decision point defined by our theorem is not only 1) a noncooperative equilibrium and 2) a profit frontier point but also 3) unique. *Hence it is a perfect candidate for tacit collusion*. In these circumstances it is not important that prices less than p^o may also define noncooperative equilibria. (In fact *every* price between $p^u/(1 - 1/n)$ and p^o will define a noncooperative equilibrium if demand is inelastic for every $p < p^o$ since then $pD(p)$ is increasing up to p^o and (46) true for every $p < p^o$.) No seller has anything to loose by quoting p^o initially, since he can always match a price cut. And equipped with "consistent conjectures" (or rational expectations) of rivals' reactions upon price cuts,[9] there is no incitement to initiate a price cut (when demand is inelastic).

While excess supply prevents price increases – and in particular prevents prices from approaching the monopoly price when firms are free to produce what they like at the ruling market price – it does not necessarily imply price decreases. If price decreases appear unavoidable to a profit-maximizing firm it is because aggregate demand is elastic at the current market price, not because excess supply initiates "price competition".

9. *The case of elastic demand*

When demand is elastic and (44) invalid, a single-price equilibrium may not exist because price *increases* may appear individually profitable at the potential candidate, which with elastic demand must be close to c. For given a market price p *below* $p^u/(1 - 1/n)$, contradicting (47), not every consumer can turn to rivals selling at p if i quotes $p_i > p$, and if p is sufficiently low, or the individual contingent demand curves sufficiently inelastic, complications may arise. To see this we note that instead of (40) we now have, for $c \leq p \leq p^u$,

$$\delta\pi_i/\delta p_i = (p - c)x_i'(p) + q_i^e \text{ at } p_i = p. \tag{48}$$

At $p = c$ we unavoidably have $\delta\pi_i/\delta p_i > 0$, as in section 5, but otherwise the elasticities of the individual contingent demand curves matter. Formally a Nash equilibrium cannot exist for $p = c$, and may not exist for $p > c$, merely because price *increases* may appear individually profitable. But for $p > c$ the risk for upward price drift and "Edgeworth fluctuations" should perhaps not be exaggerated, particularly not for the intermediate case

$$p^u < p < p^u/(1 - 1/n) \tag{49}$$

involving a discontinuous decline in sales for i when quoting $p_i > p$ (even if not *every* consumer is instantaneously lost).

Hence it is, after all, more likely than not that p^o will be a Nash equilibrium even for

$$c < p^o < \frac{p^u}{1 - 1/n} \tag{50}$$

since, moreover, (46) is still valid for $p = p^o$. But note that for

$$c < p^o < p^u \tag{51}$$

p^o will *not* lie on the profit possibility frontier, since when $p^o < p^u$ the optimal cooperative price (with free output adjustment) is higher, namely $\min(p^u, p^m)$, according to section 4.[10]

An immediate question is whether $\min(p^u, p^m)$ can be a noncooperative equilibrium too and hence the perfect candidate for tacit collusion when $p^o < p^u$. The answer is no. To see why, consider a uniform market price p $\leq p^u$. At such a price we have seen (in section 3) that $Q^e(p) = D(p)$. A price cut $p_i < p$ will be matched, implying aggregate demand $D(p_i)$, but taking $q_j = q_j^e(p)$ as given, producer i will be able to increase his market share. If q_i is adjusted to clear the market,[11] the contingent profit function will be

$$\pi_i(p_i) = (p_i - c)q_i = (p_i - c)(D(p_i) - Q^e(p) + q_i^e(p)). \tag{52}$$

We find, after some calculation, that

$$\delta\pi_i/\delta p_i = Q^e(p)(\alpha_i^e(p) - (1 - c/p)\eta(p)) \text{ at } p_i = p. \tag{53}$$

Substituting $\alpha_i^e(p) = 1/n$ according to the assumption introduced in section 3 for $p \leqslant p^u$ we see that price cutting will be attractive if

$$\eta(p) > \frac{1/n}{1 - c/p}. \tag{54}$$

From footnote 5 we find that

$$\eta(p^m) = \frac{1}{1 - c/p^m} \tag{55}$$

so p^m cannot be a noncooperative equilibrium. Substituting $p^u = c/(1 - 1/n)$ in (54) we find that p^u cannot be a noncooperative equilibrium if

$$\eta(p^u) > 1. \tag{56}$$

Since $p^o < p^u$ by assumption and $\eta(p^o) \geqslant 1$ this certainly applies if $\delta D/\delta p$ is constant on (c,p^u) and thus the demand elasticity $\eta(p)$ an increasing function of p on that interval. In that (not very restrictive) case we also see that the proper candidate for tacit collusion when $p^o < p^u$ is defined implicitly by the equation

$$\eta(p) = \frac{1/n}{1 - c/p}, \tag{57}$$

according to (53). Note that this price is higher than p^o when $p^o > c$ and thus $\eta(p^o) = 1$, since $1 - c/p < 1/n$ for $p < p^u$, including $p = p^o$. Obviously it is also higher than p^o when $p^o = c$ (and $\eta(c)$ finite). But (57) also indicates that noncooperative pricing when $p^o = c$ approaches marginal cost pricing with an increasing demand elasticity or an increasing number of firms.

We conclude by emphasizing the important analytical distinction between the case of elastic demand ($p^o < p^u$) and the case of inelastic demand ($p^o > p^u$). When oligopolists face an elastic industry demand curve the equilibrium price should be close to the competitive level (and always below p^u), as indicated above. But there are several complications. Firstly we have seen that the market shares are not uniquely determined by Nash equilibrium for prices below p^u (section 3). Secondly there is a certain risk for upward price drift ("Edgeworth fluctuations") due to inelastic individual contingent demand curves. And thirdly a noncooperative equilibrium on the profit possibility frontier is excluded.

The situation is much clearer – and in fact oligopolistic behaviour completely determinate – when demand is inelastic ($p^o > p^u$). Then the revenue-maximizing price p^o is not only a noncooperative equilibrium but also a profit frontier point. The corresponding market shares are also uniquely determined by Nash equilibrium and the risk for Edgeworth fluctuations are eliminated – at least if (44) is valid – through excess supply.

Chapter 4

Noncompetitive equilibria in atomistic markets*

1. Introduction

Fundamental to economics is the notion of a "market price" determined by "supply and demand". To give substance to this proposition when price-setting is decentralized to profit-maximizing firms one must prove 1) that independent firms quote the same price and 2) that this price is determined by the market's demand and supply curves. This we shall do in the present chapter for *atomistic* industries, defined as industries where every firm is too small to perceive any influence on aggregate output. But we shall also see that the market price is not necessarily determined by *equality* between demand D(p) and supply S(p) unless $p^o = p^c$, where p^c clears the market $D(p^c) = S(p^c)$, and p^o maximizes collective sales revenue pD(p) for $p \geq p^c$. If $p^o > p^c$ the equilibrium price will be p^o (corresponding to unit demand elasticity) and thus noncompetitive. The corresponding rate of excess supply in equilibrium will be $1 - d(p^o)$, where $d = d(p)$ is defined implicitly by the equation

$$D(p) = S(pd)d. \tag{1}$$

The possibility of noncompetitive equilibria in atomistic industries was first suggested by Chamberlin.[1] But for him "monopolistic competition" was inextricably linked to the concepts of a differentiated product and a separate market for each seller.[2] The possibility of noncompetitive equilibria in atomistic markets for homogeneous goods was explicitly denied: "the price which equates supply and demand establishes itself under pure competition because it is the only one which is consistent with maximum profits for every seller in the market" (1962 pp. 18-19).

Within the "imperfect" information paradigm the notion of a noncompetitive equilibrium was first suggested by Diamond (1971) and further developed by e.g. Hey (1974), Axell (1977) and Albrecht, Axell and Lang (1986).[3] But while noncompetitiveness appears almost inevitable with a search theoretic approach, it is not unconditional in our theory of price adjustment: it hinges on the elasticity of the market demand curve.

Here we return to the classics and postulate homogeneous goods as well as perfect information and negligible adjustment costs. Thus we assume that producers as well as consumers have perfect information on

the prices set by every firm. We also assume that every producer knows the *aggregate* demand curve and the *aggregate* supply curve. But information on *individual* demand functions and *individual* cost functions will not be needed. In other words we shall find that in atomistic industries complete knowledge of other agents' payoff functions are not needed for an individual's rational price and output decisions, in contrast to the information requirements in noncooperative oligopoly theory.[4] In fact the concept of an individual demand curve will not be used at all, in contrast to the usual approach in the price-setting literature following Hotelling (1929), Chamberlin (1962), Arrow (1959) and Shubik (1959). Instead a price-setting agent will take the aggregate demand curve as given.

Excess supply equilibria in homogeneous and atomistic markets are usually thought to be excluded by price competition. The traditional argument is that a slight price reduction will always appear profitable to a price setter *who takes his rivals' prices as given*, since it will give rise to a *discontinuous* increase of individual sales. This well-known discontinuity (due to Bertrand) is the cornerstone of the market-clearing paradigm. It implies competitive price cutting leading to a competitive equilibrium. The present chapter is a reconsideration of the rationality of such behaviour. When competitive pricing exists, it is not due to price competition but to an elastic market demand curve or, as adumbrated in the concluding section, liquidity constraints.

2. *Assumptions*

In his recent survey (1982) of models of the price mechanism, Frank Hahn summarized: "It is clear that in this area the assumptions are everything and there is a lack of primitive postulates from which the behaviour of the system could be deduced." Paradoxically enough our results depend ultimately on perfect price flexibility. Thus we assume that firms are free to change their (publicly announced) prices at any time (with negligible costs). Contemplating a price change a firm must consequently reckon with responses evoked by such a change. Hence a firm's pricing strategy cannot be represented by a single number – its (initial) book price – but must include a *response function* characterizing its reactions to its rivals' price setting behaviour.

In equilibrium a firm's pricing strategy must consequently be optimal against other firms' *complete* pricing strategies, including not only their expected initial prices but also their expected price response functions. Moreover, in a noncooperative equilibrium such expectations must be *rational*, as emphasized not least by Johansen (1982), including, in particular, rational expectations of rivals' reaction functions. The plan of the present chapter is to construct a noncooperative equilibrium conditional on perfect information of the aggregate demand and supply curves, where a noncompetitive price is supported by price response functions

modelling threats to match price cuts.

The original idea – the noncompetitive consequences of threats to match price cuts – is due to Chamberlin (1962 ch. 3) and since then frequently used (informally) in the literature on industrial organization (see e.g. Scherer 1980 ch. 5). Farm and Weibull (in chapter 2 above) offer a formalization of this idea in a model with fixed supply. Chapter 3 extends the model to include production in an oligopolistic industry with constant returns to scale and an arbitrary (but given) number of firms (excluding entry). In the present chapter we consider firms with arbitrary cost functions (subject to decreasing returns though) and we do not exclude entry. But to obtain explicit solutions in this case we assume that firms are "small".

It will be necessary to model rationing in excess supply situations. With decentralized price decisions by independent producers with (in general) different cost functions there is nothing a priori to guarantee a uniform price p. In the rest of this section, however, we shall derive a formula for expected sales on this provisional assumption.

Markets may be organized in different ways. In auction markets or with production to orders excess supply is excluded by the institutional setting. Here, however, we shall consider markets where *production precedes sales*, as is the case in most consumer markets. To fix ideas we assume that output q_i is produced before the market period and then brought to a market-place to be sold during the market period at the market price p. In the present section we take not only the (uniform) price as given but also outputs, and ask ourselves what individual sales r_i will be, conditional on outputs q_i, a uniform price p and the given aggregate demand $D = D(p)$.

In general the outcome of an unregulated matching process involving interactions between buyers and seller cannot be predicted with certainty. We consequently interpret individual sales r_i as *random variables*. Since a firm cannot sell more than it supplies we have $r_i \leq q_i$ and hence $\Sigma_i r_i \leq Q$, where Q denotes aggregate supply ($\Sigma_i q_i$). Moreover, assuming *voluntary exchange*, aggregate sales cannot exceed aggregate demand, $\Sigma_i r_i \leq D$. It follows that $\Sigma_i r_i \leq \min(Q,D)$. Assuming in addition that the market is *frictionless* we obtain

$$\Sigma_i r_i = \min(D,Q), \tag{2}$$

which is the well-known *minimum condition* of disequilibrium theory (see e.g. Muellbauer and Portes 1978).

Next we define $a_i \geq 0$ by the relation

$$Er_i = a_i q_i, \tag{3}$$

where Er_i denotes the expected value of the random variable r_i. Obviously $a_i \leq 1$ since $r_i \leq q_i$. a_i is determined by the market process. Assuming frictionless markets, $a_i = 1$ if $Q \leq D$. When $Q > D$ there is rationing. To characterize rationing by the invisible hand we postulate:

ASSUMPTION. (Anonymity). $Er_i/q_i = Er_j/q_j$ for every i and j if the price is uniform ($p_i = p$).

$Er_i/q_i < Er_j/q_j$ would imply that a unit of supply from j had a higher probability of being sold than a unit from i, in spite of postulated product homogeneity and price uniformity (and absence of advertising). Excluding such biases amounts to excluding discriminatory marketing technologies, like a systematic underexposure of the products from one particular firm.

Substituting $Er_i = a_iq_i = aq_i$ in (the expected value of) the min-condition we obtain $\min(D,Q) = \Sigma_i aq_i = aQ$ and hence $a = \min(1,D/Q)$.

This completes the proof of the following result:

LEMMA 1. (Expected sales). $Er_i = q_i\min(1,D/Q)$ in a frictionless and anonymous market where the product is homogeneous and the price uniform.

In deriving this formula (generalizing the proportional rationing scheme to a stochastic setting) we do not *assume*, as Green (1980) and Weinrich (1984) do, that the only parameters affecting the probability distribution of r_i are q_i, D and Q. Weinrich generalizes Green's result by *dropping* Green's anonymity requirement; here we *reformulate* Green's anonymity requirement and make it the basis of our characterization of rationing by the invisible hand.

3. *Effective supply*

We shall derive our market price theorem in three steps. In this section we consider a fixprice economy where every firm takes a noncompetitive price ($p > p^c$) as exogenously given. In the next section we assume that the market price is chosen by firms through voting (cooperative pricing). Thus firms decide individually on outputs but collectively on the market price. We find that every firm prefers p^o to be the market price, independent of its cost function, so the vote will be unanimous. And finally, in section 5, we shall see that this price is indeed supported by noncooperative behaviour when firms are free to change their prices at any time.

Define S(p) in the usual way as aggregate competitive supply, i.e. $S(p) = \Sigma_i s_i(p)$, where individual supply $s_i(p)$ is derived on the assumption that everything produced will be sold. But this assumption is not consistent with a noncompetitive price ($p > p^c$). Suppliers with a "disequilibrium awareness" (Fisher 1983) will realize this and adjust their supplies accordingly. To derive a firm's adjusted or *effective supply* we assume that every firm wishes to maximize the expected value of its profits

$$pEr_i - c_i(q_i). \tag{4}$$

There are four implicit assumptions behind this formulation. Firstly we assume that the market period is so short that time preferences (discounting) can be neglected. Revenues arriving at the end of the period are no less valued than revenues arriving at the beginning: only total revenues pr_i matter. Of course this is a convenient implication of every *single period* analysis.

Secondly we assume that goods are *perishable* so that inventories remaining at the end of the market period are without value. This assumption is introduced for simplicity but also without restriction in the present context. For if an excess supply situation is found to be supported by producers' self-interest when remaining stocks are worthless, the support can only be strengthened when an inventory evaluation function is added to the payoff function.

Thirdly, maximization of *expected* profits presupposes risk neutrality. Alternatively we may assume that the market period, even if it is short enough to make time preferences negligible, is long enough (say a year) to enable a representation of (yearly) sales as a sum of more or less independent (weekly) components. In that case we may invoke the (strong) law of large numbers to rationalize the assumption

$$Er_i = r_i \text{ (with probability 1)}, \tag{5}$$

which makes our model deterministic and the usual profit function

$$pr_i - c_i(q_i) \tag{6}$$

relevant. But it might be objected that "small" firms cannot ignore stochastic variations of individual sales in excess supply situations, or in other words that assumption (5) is sometimes too strong for "small" firms. In that case we fourthly assume – apart from risk neutrality – that profit maximization is not restricted by liquidity constraints. The consequences of relaxing this assumption will be discussed in the concluding section.

Combining (4) with lemma 1 *or* (6) with lemma 1 and (5) we obtain, in both cases,

$$\pi_i = pD(p)q_i/Q - c_i(q_i) \tag{7}$$

as the firm's payoff function for a noncompetitive price ($p > p^c$).

Now, when deciding on individual output q_i a producer can take the price p as given (by assumption in this section) and he can also take the aggregate demand D(p) as given. But he cannot take aggregate output $Q = \Sigma_j q_j$ as given when choosing q_i, since Q is influenced by other firms' outputs, chosen simultaneously and without cooperation or communication. We conclude that firms are facing a noncooperative game in quantities.

Suppose there are n firms and introduce the market shares

$$\alpha_i = q_i/Q, \; i = 1,2,\ldots n, \tag{8}$$

and the rate of excess supply $1 - d$ where

$$d = D(p)/Q. \tag{9}$$

Differentiating the payoff function (7) we obtain

$$\delta\pi_i/\delta q_i = p(1 - \alpha_i)d - c_i'(q_i) \tag{10}$$

and note that $q = (q_i)$ will be an interior ($d < 1$) Cournot-Nash equilibrium if

$$p(1 - \alpha_i)d = c_i'(q_i),\ i = 1,2,...n. \tag{11}$$

As emphasized not least by Johansen (1982) an individually rational decision in a noncooperative decision situation is part of the solution to such an equation (if unique). But how can an individual producer solve this equation unless he knows the cost function of every other firm? Game theory usually relies on the principle of complete information, implying knowledge of every player's payoff function. Here we assume instead that firms are small:

ASSUMPTION. (Atomistic firms). Every firm's market share is negligible.

In this case a producer need not know other producers' cost functions to be able to solve (11). For substituting $\alpha_i = 0$ in (11) we obtain

$$pd = c_i'(q_i) \tag{12}$$

or, equivalently,

$$q_i = s_i(pd). \tag{13}$$

Hence $Q = S(pd)$ and $d = D(p)/Q = D(p)/S(pd)$. We conclude that the Cournot-Nash equilibrium is defined by

$$q_i^e = q_i^e(p) = s_i(pd(p)) \tag{14}$$

where $d(p)$ is the (unique) solution to the equation

$$D(p) = S(pd)d. \tag{15}$$

Note that $d(p) < 1$ for $p > p^c$ so that (14) is indeed an interior solution (excluding market-clearing) for every $p > p^c$.

We interpret $1 - d(p)$ as *rational expectations* of the rate of excess supply at p, with reference to the standard interpretation of a noncooperative equilibrium (see e.g. Johansen 1982) as well as the following illustrative short-cut to equation (15).

Starting from expectations $1 - d^e$ of the rate of excess supply $1 - D/Q$ a producer which neglects his own influence on D/Q should behave competitively with respect to the "effective" price pd^e, according to (7), i.e. his supply should be $s_i(pd^e)$. But then $Q = S(pd^e)$. Consistency requires

$$D(p)/S(pd^e) = d^e, \tag{16}$$

so that $d^e = d(p)$ in expectational equilibrium, where $d(p)$ is defined implicitly by (15).

Multiplication of (15) with p yields

$$pD(p) = S(pd(p))pd(p) \tag{17}$$

and we obtain the following useful result:

LEMMA 2. Suppose that $S(p)$ is an increasing function of p and let $1 - d(p)$ denote rational expectations of the rate of excess supply according to (15). Then $pd(p)$ increases if and only if $pD(p)$ increases.

As an illustrative example, consider the linear schedules

$$D = W - a(p - p^c),\ S = W + b(p - p^c), \tag{18}$$

where $W = D(p^c) = S(p^c)$ and a and b are positive constants. Then we find, after some calculation, that

$$2pd(p) = p_0 + (p_0^2 + 4pD(p)/b)^{1/2}, \tag{19}$$

where p_0 is defined by $S(p_0) = 0$.

As a first application of lemma 2 we note that the aggregate *effective supply curve*

$$Q^e = S(pd(p)) \tag{20}$$

will be backward-bending at p when the usual competitive supply curve is forward-bending if aggregate demand is elastic at p, $-pD'(p)/D(p) = \eta(p) > 1$, since $(pD)' = D(1 - \eta)$. If demand is elastic for every $p > p^c$, effective supply will be less than $D(p^c)$ for every $p > p^c$. If demand is inelastic at p^c, effective supply will normally be increasing up to p^o and then decreasing.

4. *Cooperative pricing*

So far the market price has been exogenously given. In the next section firms will decide individually on prices as well as outputs, without cooperation or communication. In the present section an intermediate case is discussed. Thus firms decide individually on outputs but collectively on the market price.

To fix ideas, imagine that a statistician carries out two surveys referring to the impending market period. One questionnaire is posted to consumers, asking for demand at various prices on the hypothetical assumption that everything demanded can be purchased. Another questionnaire is posted to producers, asking for supply at various prices on the hypothetical assumption that everything supplied can be sold. By summing over consumers and producers respectively, the statistican ascertains the demand curve $D(p)$ and the supply curve $S(p)$ for the next market period. These aggregate data – and no individual data – are published in a bulletin.

Given this information the market price is then determined by firms through voting. (Since the vote will be unanimous, as we shall see below, the specification of the voting procedure is immaterial.) This price will be announced by the statistician as the ruling market price for the current market period. It cannot be changed until the next market period.

Having determined the market price, firms produce what they like at the market price agreed upon. Outputs are then brought to a market place where consumers buy what they like, taking the ruling market price as given (excluding haggling and bargaining).

Given these rules of the market game the market price will be p^o, maximizing $pD(p)$ for $p \geq p^c$. To see this we shall use backward induction (as in dynamic programming). Recall from section 3 that individual profit in equilibrium at p *after* quantity adjustment is given by the "indirect" profit function

$$\pi_i(p) = pd(p)q_i^e - c_i(q_i^e) \tag{21}$$

where $d(p)$ is defined implicitly by equation (15) and $q_i^e = s_i(pd(p))$. Hence q_i^e maximizes $\pi_i = pd(p)q_i - c_i(q_i)$ and it follows from the envelope theorem that

$$d\pi_i(p)/dp = \delta\pi_i/\delta p = (pd(p))'s_i(pd(p)). \tag{22}$$

From (22) and lemma 2 it follows that every firm will prefer p^o to be the market price *irrespective of its cost function*. The vote on p^o will consequently be unanimous.

Alternatively we may argue as follows. A firm with a disequilibrium awareness should realize that its profit for $p \geq p^c$ will not be $pq_i - c_i(q_i)$ but $pd(p)q_i - c_i(q_i)$, where $pd(p)$ denotes average revenues per unit of supply (in equilibrium after quantity adjustment). Moreover, an "atomistic" firm is too small to perceive any influence from q_i on $d(p)$. Hence it quite naturally wishes to maximize the "effective price" $pd(p)$, irrespective of its output and cost function. But maximization of $pd(p)$ turns out to be equivalent to maximization of the industry's collective sales revenues $pD(p)$. Hence every firm votes for p^o and produces $s_i(p^od(p^o))$, thus behaving competitively with respect to the *effective* price $p^od(p^o)$.

5. *Noncooperative pricing and production*

It remains to discuss pricing and production when there is no cooperation or communication at all between firms. A producer has to decide on which output to produce and bring to the market place. He also has to choose an initial (list) price to quote when the market opens and, moreover, a pricing strategy to guide his reactions when he can observe his rivals' prices. This decision problem is truly noncooperative, since a producer has to choose output, initial price and pricing strategies without knowing outputs, initial prices and pricing strategies chosen simultane-

ously by other firms.

In section 4 as well as in section 3 a producer could take the market price as given when choosing output, but here this is no longer possible. Furthermore, there are no *exogenous* market price expectations to take as given in our model, like, for example, the "prevailing industry price" of Phelps and Winter (1970 p. 314), Negishi's "current market price" (1979 p. 89) or Iwai's "index of the general price level" (1981 p. 22). In our model of decentralized price-setting expectations of the market price must be *endogenous* (or "rational" or "consistent"), as part of a non-cooperative equilibrium calculated independently by every firm.

All that a firm can take as given is the market demand curve and the industry supply curve, apart from the market structure (atomistic, frictionless and anonymous) and the presumption that every other firm is a rational profit-maximizer. A producer's problem is to develop a proper decision theory conditional on this information. We assume that every producer has come as far in doing this as we now have.

When market demand is elastic and $p^o = p^c$ it follows immediately that competitive pricing ($p_i = p^c$) must be the best price policy available to atomistic firms. For on one hand not even cooperative pricing would yield a higher price (section 4). And on the other hand it is easy to see that individual prices less than the market-clearing price ($p_j \leq p^c$ for every j and $p_i < p^c$ for some i) cannot be in equilibrium.[5]

We turn now to the case of inelastic demand and $p^o > p^c$. According to the market-clearing paradigm a noncompetitive market price such as p^o is excluded by price competition. The well-known text-book argument (following Bertrand) is that $p_i = p > p^c$ cannot define an equilibrium since every firm perceives a possibility to increase its sales discontinuously by choosing a price slightly less than p. Formally $p_i = p$ is not optimal against $p_j = p$, $j \neq i$, for any $p > p^c$, proving that $(p_i) = (p)$ cannot be a Nash equilibrium for $p > p^c$.

But it does not follow that the market-clearing state is an equilibrium point. To see this we first observe that a firm's complete strategy in a price-quantity game consists of two numbers, $\sigma_i = (p_i,q_i)$. The market-clearing paradigm suggests that $\sigma = (\sigma_i)$ is an equilibrium point for $p_i = p^c$, $q_i = s_i(p^c)$ and $\Sigma_i s_i(p^c) = D(p^c)$. To check this we take $\sigma_j = (p^c,s_j(p^c))$ as given for $j \neq i$ and contemplate strategies $(p_i,q_i) \neq (p^c,s_i(p^c))$ for firm i. Then consumers will buy $D(p^c) - s_i(p^c)$ (but no more!) from i's competitors at p^c and the "contingent" demand $q_i = q_i(p_i)$ from i at $p_i \geq p^c$. With

$$\pi_i = p_i q_i(p_i) - c_i(q_i(p_i)) \tag{23}$$

we now always have

$$\delta\pi_i/\delta p_i = s_i(p^c) \text{ at } p_i = p^c, \tag{24}$$

so σ cannot be a noncooperative equilibrium unless $s_i(p^c) = 0$ for every i, contradicting the assumption that $\Sigma_i s_i(p^c) = D(p^c) > 0$. We conclude that

a correct ending of the Bertrand story is that no equilibrium exists.

It is not always appreciated, however, that the Bertrand story crucially depends on *rigid prices*. Only in this case, when firms are irrevocably committed to initial price quotations (as in sealed bidding), can a pricing strategy be represented by a single number. This is not possible in an industry where firms are free to observe and change their prices at any time. In this setting a firm's *market strategy* σ_i consists of a *response function* f_i in addition to its initial price p_i and output q_i,

$$\sigma_i = (p_i, q_i, f_i). \tag{25}$$

f_i prescribes a firm's price at time t (after the beginning of the market period) as a function of past behaviour. In general this includes every firm's pricing behaviour up to t (as well as outputs).

The no-response function $p_i(t) = p_i$ is but a special case. Of course σ (with components σ_i) cannot be an equilibrium point for $(p_i) = (p^o)$ with such response functions. For taking $p_j(t) = p^o$ as given for every $j \neq i$, a slight price cut $p_i < p^o$ will appear attractive to i. But it does not follow from this argument that it will be rational for i to initiate a competitive process leading to p^c. Instead we conclude that i's decision theory cannot be in equilibrium (cf. e.g. Luce and Raiffa 1957 p. 63). In particular his predictions of competitors' behaviour must be revised.

Suppose, to begin with, that he goes from the price taking hypothesis to the price cutting hypothesis, predicting that other firms are price cutters with price-response functions

$$p_i = \max \{ p^c, \min \{ p_j \} - \varepsilon \} \tag{26}$$

for some (small) $\varepsilon > 0$. When prices $\{ p_j \}$ are observed a price cutter consequently reacts by slightly underbidding the lowest bidder (subject to the obvious restriction that he never goes below p^c). But a firm confronted with price cutters can do no better than being a price cutter himself, for otherwise he risks losing all his sales. Assuming perfectly flexible prices, so that response times can be ignored, the trading price will then in effect be equal to p^c, irrespective of initial list prices. It follows that market strategies with price-response functions according to (26) are in equilibrium for $q_i = s_i(p^c)$ and any initial prices $p_i \geq p^c$. The competitive story is indeed compatible with a certain noncooperative equilibrium, even if it does not rely on the price taking hypothesis but on the price cutting hypothesis. But we cannot conclude that rational firms will behave competitively. For there may be other equilibria, as in other sequential games (see e.g. Friedman 1983).

Following Sweezy (1939) and Marschak and Selten (1978) we now introduce the "kinked" response function

$$p_i = \max \{ p^c, \min \{ p_j \} \} , \tag{27}$$

– or min p_j for short – and consider the market strategies

$$\sigma_i^k(p) = (p, q_i^e(p), \min p_j), \tag{28}$$

implying that every firm quotes p initially and produces the corresponding effective supply $q_i^e(p)$ and then behaves according to the kinked response function (matches every price cut instantaneously). Let $\sigma^k(p)$ denote the corresponding vector (decision point).

THEOREM. $\sigma^k(p)$ is an equilibrium point for $p > p^c$ if p maximizes $p'D(p')$ on the interval $p^c \leq p' \leq p$.

To see this we consider a producer i and let him take his rivals' *complete* pricing strategies as given – not merely their initial prices $p_j = p$ – when contemplating defections $p_i \neq p$ and $q_i \neq q_i^e(p)$. For $p_i < p$ the trading price will then be p_i – and not p – due to the kinked response functions. Hence the producer's (contingent) profit function will be

$$\pi_i(p_i, q_i) = p_i D(p_i) q_i / \Sigma_j q_j - c_i(q_i), \; p_i < p, \tag{29}$$

where $q_j = q_j^e(p)$ for $j \neq i$. Next we note that no pair (p_i, q_i) such that $p_i < p$ can be profit-maximizing if

$$p_i D(p_i) < pD(p) \text{ for every } p_i < p, \tag{30}$$

since then (p, q_i) is more profitable than (p_i, q_i), according to (29). But (30) is true by assumption. Thus price cutting cannot be attractive. To complete the proof we observe that with $p_i = p$ it will be optimal to produce $q_i^e(p)$, since $q^e(p)$ is a Cournot-Nash equilibrium by construction.[6]

Normally pD(p) will be increasing up to p^o and then it follows from our theorem that every p such that $p^c < p \leq p^o$ will define an equilibrium. This lack of uniqueness suggests that it might be difficult for firms to reach tacit collusion without communication. But note that given $p_j = p < p^o$, $j \neq i$, $p_i = p^o$ gives the same outcome to i as $p_i = p$. In other words a firm has nothing to loose by quoting p^o initially, since it can always match a price cut. Realizing this every firm will quote p^o, yielding higher individual profits to every firm than any other trading price, according to section 4.

The monopoly price p^m – higher than p^o since a monopolist operates where the demand elasticity exceeds unity – could, in theory, involve still higher profits for every firm, but only if the costs of organizing a production cartel and a system for division of profits were negligible. With prohibitive costs – or a prohibitory law – p^o stands out as the "second best". We emphasize that when firms are free to produce what they like at the ruling market price, the "first best" is not attainable. Excess supply does not entail price *decreases* – when demand is inelastic – but it does prevent prices from *increasing* to the monopoly price.

The rate of excess supply in equilibrium at p^o is not determined by the *competitive* supply curve but the *effective* supply curve (section 3). Recalling that the effective supply curve is vertical at p^o we realize that the equilibrium rate of excess supply is less than what is suggested by the classical formula $1 - D(p^o)/S(p^o)$. The exact formula is $1 - d(p^o)$, where

$d = d(p)$ is defined implicitly by the equation $D(p) = S(pd)d$.[7] In equilibrium a producer will experience a discrepancy between supply $s_i(p^od(p^o))$ and sales $d(p^o)s_i(p^od(p^o))$, but of course he cannot interpret this as an additional constraint on his behaviour. He may regret inventories left over at the end of the market period as a waste of resources, but lowering the price to get rid of them would also lower his profits.

Entry is *not* excluded in the present model. The *competitive* supply curve reflects potential output from every potential firm. The *effective* supply curve reflects endogenous output restriction, including exit. A firm's output will be positive at p^o if and only if $s_i(p^od(p^o)) > 0$. What matters to a rational potential entrant is not the nominal price p^o, but the *effective* price $p^od(p^o)$.

6. *Competitive equilibria due to liquidity constraints*

Liquidity constraints may enforce competitive pricing even when the industry demand curve is inelastic. To see this we consider the problem

$$\max_{p_i,q_i} p_iEr_i - c_i(q_i) \tag{31.1}$$

$$\text{s.t. } Pr(p_ir_i + m_i \geq c_i(q_i)) = 1. \tag{31.2}$$

Here we have assumed that the producer's utility is a linear function of money but this we do for simplicity only. What is important in the present context is the liquidity constraint (31.2), saying that with probability one the producer must be able to pay for current production out of current sales revenue (p_ir_i) and initial money holdings (m_i). This means on one hand that credit may be extended to the end of the current period or year but on the other hand that loans between years are excluded. Thus there is implicitly a capital market in our model *but it is not perfect*.

When $Er_i = r_i$ with probability one problem (31) reduces to the conventional one. But this deterministic assumption is legitimate – when there is excess supply – *only if* yearly sales r_i may be represented as a sum of more or less independent monthly or weekly sales (in which case we may invoke the law of large numbers). Now, when a firm becomes smaller such a representation may become suspect. Small firms cannot always ignore stochastic variations of sales in excess supply situations, since even if $p_iEr_i - c_i(q_i) > 0$ it may happen that $p_ir_i - c_i(q_i) + m_i < 0$ with some positive probability. In this case a producer may have to choose between 1) a noncompetitive price with positive expected profits (when aggregate demand is inelastic) but a nonnegligible risk of bankruptcy and 2) a competitive price with no risk of bankruptcy but zero expected profits.

A producer may feel that "there should be a law against such dilemmas" but faced with one he is forced to quote the competitive price. Note that the risk of bankruptcy is not due to uncertainty (meaning that prob-

abilities of specific outcomes are unknown) but to the stochastic nature of a matching process in an anonymous market where $Er_i = dq_i$ (cf. lemma 1), and, in general, the probability distribution of r_i determined by d, even if the *form* of the probability distribution depends on the character of the good and the institutional setting.[8]

The variability of individual sales of small firms in excess supply situations may of course be eliminated through a production cartel, but excluding such institutions, as well as perfect capital markets, we predict that atomistic firms will quote a competitive price even when aggregate demand is inelastic, provided that, in addition, their liquidity reserves (m_i) are strained.

The introduction of *product differentiation* may, on the other hand, reduce the variability of individual sales through the creation of a stable, separate market for each seller (cf. Chamberlin 1962). In this case a noncompetitive price (p^o) may persist even in an atomistic market with illiquid firms, provided that aggregate demand is inelastic at the competitive price.

Chapter 5

Job search and effective labour supply*

1. Introduction

Let D(w) and S(w) denote the demand and supply curves in a homogenous labour market. The usual formula for the rate of unemployment is $1 - D(w)/S(w)$. But S(w) denotes the sum of individual supply curves derived on the assumption that employment will be equal to supply and this assumption is not valid when there is unemployment. Rational workers should realize this and adjust their supplies accordingly. In this chapter I shall argue that the true rate of unemployment (in expectational equilibrium) is 1-d where $d = d(w)$ is defined implicitly by the equation $D(w) = S(wd)d$.

We assume that the risk for unemployment is equally distributed among workers in a homogeneous market. Currently employed cannot ignore the risk for future unemployment; nor can currently unemployed ignore the chance of future employment. Formally we assume that every worker expects his employment r_i to be proportional to his supply h_i, $r_i = dh_i$, where d denotes the expected degree of aggregate employment. This amounts to postulating *proportional rationing* when there is excess supply. In section 2 I shall give a rationale for this assumption.

In section 3 we take the wage as exogenously given and derive the corresponding "effective" labour supply from workers with a "disequilibrium awareness" (cf. Fisher 1983) and, at the same time, the corresponding (true) rate of unemployment. Our "disequilibrium" approach also suggests a simple solution to the well-known paradox that a rise in the demand for labour may increase its supply.

Our formulas depend on the simplifying assumption that a worker has a purely *instrumental* attitude towards time spent in market activities, so that the division of this time between work and job search does not matter in itself, only the total payoff in terms of goods. But the consequences of a preference for work to search (or vice versa) are also indicated, in an appendix.

2. Proportional rationing

To see why rationing by the invisible hand should be (approximately) proportional we first consider a simple example. Consider unemploy-

ment among dock-workers in a given harbour. Suppose labour is hired for each day separately (but the wage revised only once every year). The market place is a hiring hall where employers and workers meet every morning. Employers attempt to select the workers best suited to current tasks or simply pick those who arrive first in the morning. Each day S workers compete for D jobs ($D < S$), by argumentation or simply by queuing. The outcome of the matching process varies between days. Then we may assume that the probability of employment is $d = D/S$ every working-day for every worker, independent of previous employment decisions. Let there be T working-days in a year. It follows from the *law of large numbers* that every worker will be employed (approximately) dT days each year.

So far we have assumed that every worker visits the hiring hall every working-day, i.e. that the supply of every worker is T days in a year. But suppose, in general, that a worker's supply is h_i days, $h_i \leqslant T$. Then aggregate labour supply is $\Sigma_i h_i = H \leqslant ST$ days per year or H/T days per working-day on the average. In general supply may now vary between days, but to avoid unnecessary technicalities we simply assume that variations are smoothed out (by independence between labour supply decisions). Thus we postulate that each working-day H/T workers come to the hiring-hall, and of course we still consider the case when demand D is less than supply H/T. Then the probability of employment is $D/(H/T) = TD/H$ on each of the h_i days a worker visits the hiring hall and it follows that he will be employed (approximately) $h_i(TD/H)$ days a year. This is proportional rationing.

Formally, let h_i denote an individual's supply and Er_i the expected value of his sales (employment) r_i. To characterize rationing by the invisible hand in a homogeneous market we postulate:

ASSUMPTION. (Anonymity). $Er_i/h_i = Er_j/h_j$ for every i and j.

$Er_i/h_i < Er_j/h_j$ would imply that a unit of supply from j had a higher probability of being sold than a unit from i, in spite of postulated homogeneity. Excluding such biases the market process is *anonymous* and $Er_i/h_i = Er_j/h_j$ for reasons of symmetry.

We then recall that *voluntary exchange* in a *frictionless market* where aggregate demand D is less than aggregate supply implies

$$\Sigma_i r_i = D.$$

Writing $Er_i/h_i = Er_j/h_j = d$ it follows that

$$D = \Sigma_i Er_i = d\Sigma_i h_i = dH.$$

Hence $d = D/H$ and

$$Er_i = (D/H)h_i, \tag{1}$$

which is proportional rationing applied to *expected values* in the general case when r_i is a random variable with a nondegenerate probability distribution.[1]

Proportional rationing in the usual, deterministic sense ($r_i = (D/H)h_i$) follows immediately if, in addition, we postulate a deterministic model, i.e.

$$\Pr(r_i = Er_i) = 1. \tag{2}$$

Of course a deterministic model has its limitations and (2) may sometimes be a too restrictive assumption. But whenever r_i may be decomposed into a sum of independent random variables we may rationalize (2) by the (strong) law of large numbers. Moreover, such a decomposition of actual employment is not unreasonable, *provided the period considered is long enough*, at least one year, so that actual employment will be affected by many, independent decisions.

In the disequilibrium literature proportional rationing has been characterized as "manipulable", leading to overbidding and instability (see e.g. Drazen 1980 and Benassy 1982). To see on one hand that there *is* a problem, and on the other hand how comparatively innocuous it is, consider a worker who reports h_i/d to his trade union (as part of a work-sharing arrangement) in order to secure h_i hours of work, representing his true supply (desired hours of work). Here we assume that $1 - d$ represents that unemployment rate which corresponds to initial (true) supply. Now, if *all* workers behave like this, aggregate supply will turn out to be, not H but H/d and hence $r_i = (D/(H/d))(h_i/d) = dh_i$. Then expectations will not be verified and behaviour must be modified.

Myopic behaviour might entail further overbidding and thus initiate an unstable process, as suggested by Drazen (1980 p. 288). But this is only a theoretical possibility. It is excluded by the requirement that trade offers must be "credible", a concept emphasized by Heller and Starr (1979). In the labour market this concept is particularly relevant, since an offer to work more than full-time soon looses its credibility. But what about a worker who desires part-time? It might be thought that part-time workers would be interested in a process of overbidding, which necessarily stops at $h_i = h$ for some h (much less than 24 hours per day) and thus a uniform job allotment $r_i = D/n$. But this possibility is relevant only if D/n happens to correspond to the part-timer's true supply. As a rule D/n will exceed his true supply, necessitating a choice between 1) turning the offer down (and thus endangering his credibility), 2) accepting over-employment by himself (and simultaneous under-employment by other workers) and 3) adjusting to desired working hours by absenteeism (endangering his credibility and his job). The only way out of this dilemma is to abstain from attempts to manipulate the supply.

We conclude that even if a system of work-sharing with proportional rationing *formally* is manipulable, manipulation does not pay. It might be tempting for an individual to attempt to manipulate his supply on the assumption that other workers are honest, but such an attempt will succeed only if honesty is the rule. Thus, the *existence* of manipulation presupposes the *prevalence* of nonmanipulation.

But above all it must be realized that labour supply is normally not a signal that can be manipulated without cost. A worker does not get "his" share of employment D by simply reporting his labour supply unless there is work-sharing. Sales in a market economy presuppose supply brought to a market place. Excluding rationing by a visible hand,[2] we must consequently interpret labour supply as time actually spent in the market ("currently available for work"), either working or seeking work.

Without active search when unemployed a worker cannot count on proportional rationing through an anonymous matching process in a homogeneous market. As emphasized by Gale (1979 p. 321): "A worker who fails to obtain a job with one employer will visit other employers looking for work, rather than increase his supply offer to the first." Gale's conclusion is to interpret exaggeration as a proxy for search. The logical next step in this direction seems to be to substitute search for exaggeration altogether and model labour supply decisions in excess supply situations as a simultaneous choice of work, leisure *and search*. This we shall do in the next section, emphasizing that search in our model can *not* be interpreted as search for a higher real wage, as in traditional search models. It is simply search for a job – *any* job – at a given wage, and thus *literally* job search.

3. *Effective labour supply*

Consider the text-book problem

$$\max_{s} u_i(ws, k - s) \tag{3}$$

defining an individual's (competitive) labour supply $s_i = s_i(w)$ in hours. w denotes the (given) real wage, ws consumption goods and $k - s$ nonmarket time, devoted to homework and leisure. $k = 24l$ where l denotes the length of the current period in days. To fix ideas we may assume that the current period is a year and that labour supply decisions are revised not more than once every year.

Let w^c denote the competitive or market clearing wage defined by $D(w) = S(w)$, where D(w) denotes the demand curve and $S(w) = \Sigma_i s_i(w)$. Now (3) defines an individual's optimal supply conditional on the assumption that employment will be equal to supply. But this assumption is valid if *and only if* $w \leq w^c$. If $w > w^c$ there will be unemployment and unless workers are barred from this information, it should affect their supply decisions. When there is excess supply rational agents should develop a "disequilibrium awareness", as emphasized by Fisher (1983). In our model this concept is represented by individuals' expectations of aggregate demand per unit of aggregate supply, d. Taking d as given (in addition to w), an individual's supply h will lead to employment dh, referring to proportional rationing (section 2). Note carefully that ration-

ing in proportion to supply in a homogeneous and anonymous market presupposes that supply is actually "brought to the market place", implying in this context that h denotes time spent "currently available for work", so that time "without work" during h is devoted to "seeking work". It follows that an individual's problem now is, in general,

$$\max_{h} u_i(wdh, \bar{k} - h, dh, h - dh). \tag{4}$$

But with a purely *instrumental* attitude towards time spent in the market (h), the division of this time between employment (dh) and job search (h − dh) does not matter, so (4) reduces to

$$\max_{h} u_i(wdh, k - h). \tag{5}$$

In this section our point of departure is (5). The consequences of preferring work to search (or vice versa) is discussed in an appendix, starting from (4).

It follows immediately from (5) and (3) that

$$h_i = s_i(wd) \text{ and } H = S(wd), \tag{6}$$

where H denotes aggregate labour supply, $\Sigma_i h_i$.

Consistency requires $d = D(w)/H$ or

$$D(w) = S(wd)d. \tag{7}$$

Thus, the rate of unemployment in *expectational equilibrium* is not $1 - D(w)/S(w)$ but $1 - d$ where $d = d(w)$ is defined implicitly by equation (7). (Note that $d < 1$ unless $S(w) = D(w)$.) *Effective* labour supply is not S(w) but S(wd(w)). What matters to workers in an excess supply situation is not nominal income − what they would get at the ruling wage given full employment throughout the period − but *average earnings* (in equilibrium after quantity adjustment). Workers with a "disequilibrium awareness" behave competitively with respect to the *effective wage* wd(w), not with respect to the nominal wage w.

To characterize the effective supply curve H(w) *when the ordinary supply curve S(w) is forward-bending* we first observe that $H(w) < S(w)$ since $d < 1$. Moreover, (7) yields

$$wD(w) = S(wd)wd.$$

Since S(wd)wd is an increasing function of wd it follows that:

$$\text{wd(w) increases if and only if wD(w) increases.}^{3} \tag{8}$$

Hence H(w) = S(wd(w)) decreases if and only if

$$(wD)' = D(1 - \eta) < 0,$$

where $\eta = \eta(w)$ denotes the demand elasticity, $\eta = -wD'/D$. We conclude that the effective supply curve is *backward-bending* at w (when the

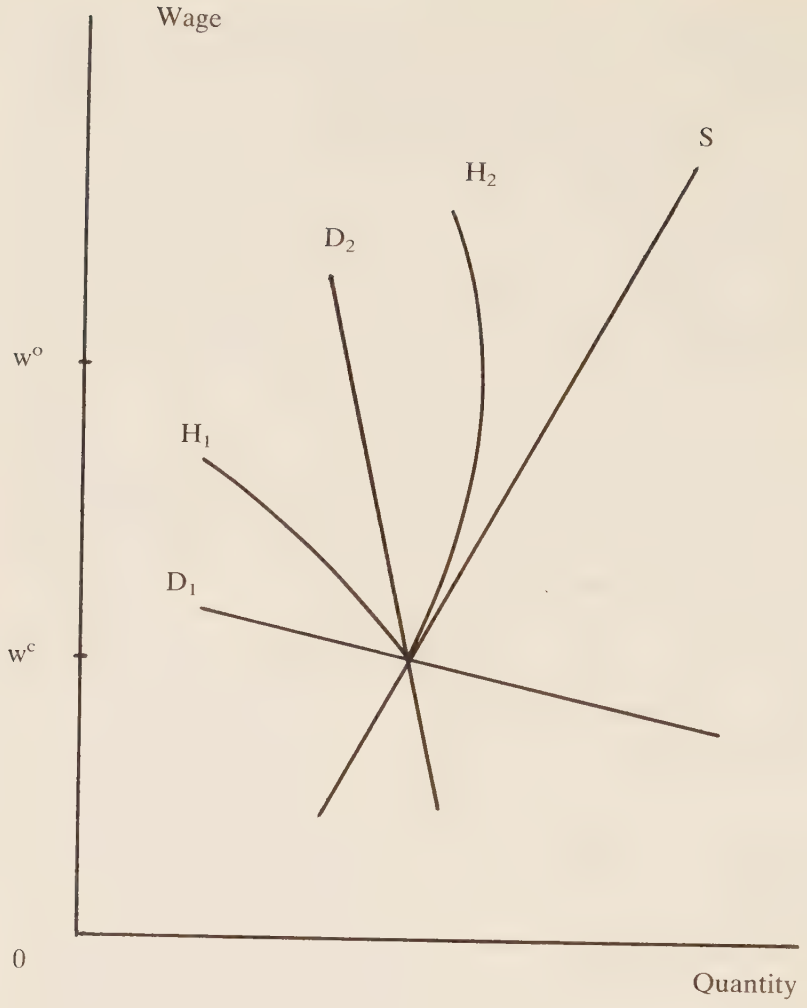

Figure 1. Effective labour supply when demand is elastic (H_1, corresponding to D_1) and inelastic (H_2, corresponding to D_2).

competitive supply curve is forward-bending) if and only if $\eta(w) > 1$.

It follows that the effective supply curve is backward-bending (when the competitive supply curve is forward-bending): 1) for every $w > w^c$ if $\eta(w^c) \geq 1$ and 2) for every $w > w^o$ if $\eta(w^c) < 1$, where w^o is defined implicitly by the equation $\eta(w^o) = 1$. Figure 1 illustrates these results. Note that these simple formulations presuppose a wage bill $wD(w)$ which has a unique maximum at w^o and is strictly increasing (decreasing) below (above) w^o. We may assume, for instance, that the labour demand curve is (approximately) linear in the neighbourhood of the market-clearing wage.

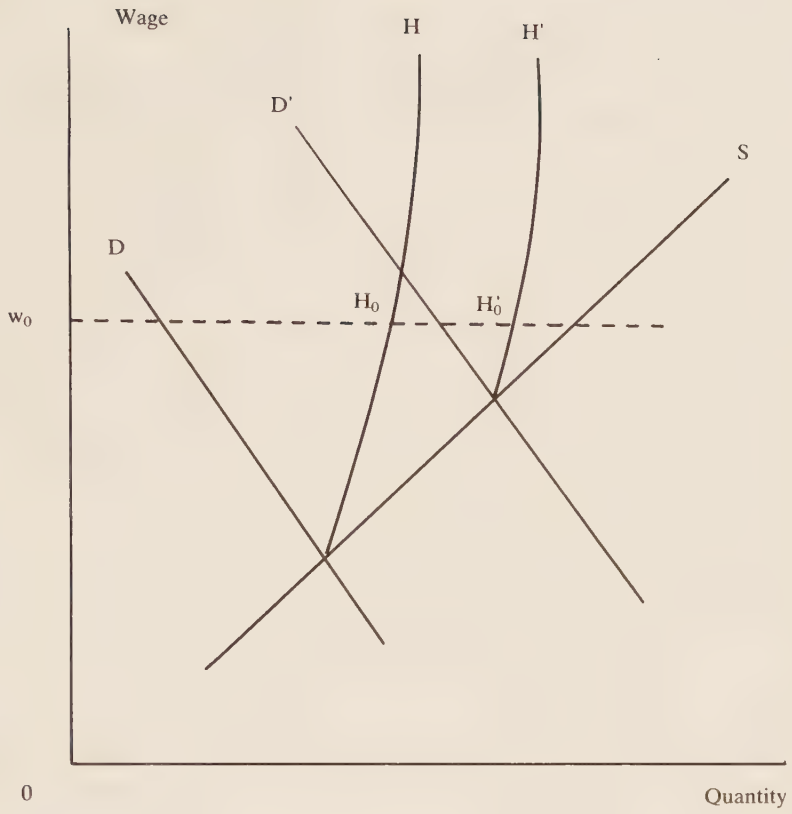

Figure 2. The added worker effect ($H_0 - H_0'$).

We conclude this chapter by first emphasizing that *observed* unemployment rates refer to *effective* supply. Thus an *observed* backward-bending supply curve is not necessarily due to a dominant income effect. It may be due to labour supply adjustments when labour demand is elastic.

Second we recall the well-known fact that an upward shift of the labour demand curve may increase observed labour supply *for a given wage*: supply appears to be affected by demand. In the present framework this "added worker effect" has a very direct explanation, since *effective* supply *is* affected by demand, even if *competitive* supply is not. Figure 2 illustrates this in the case when demand is inelastic ($\eta(w^c) < 1$). Note that an upward shift of the demand curve implies a rightward shift of the

effective supply curve if the (fixed) *competitive* supply curve is forward-bending.

Appendix
Effective labour supply in the general case

When the division of the time spent in market activities between work (dh) and search (h − dh) matters, an individual's problem is (4) or, suppressing the last argument of the utility function for simplicity and without restriction,

$$\max_{h} u_i(wdh, k - h, dh), \tag{A.1}$$

where d = d(w) is to be interpreted as aggregate demand per unit of aggregate supply *in expectational equilibrium* (to be derived below). The first-order condition is

$$u_1wd - u_2 + u_3d = 0, \tag{A.2}$$

where u_j denotes the derivative of the utility function u (suppressing the index i) with respect to its j:th argument. Hence

$$u_2/u_1 = w(1 + v)d, \tag{A.3}$$

where $vw = u_3/u_1$. We interpret u_3/u_1 as the (marginal) value (in terms of goods) of transferring one hour from search to work *in addition to* the real wage. In general v is a function of h, but suppose not (for simplicity). Then it follows from (A.3) that

$$h = s(w(1 + v)d), \tag{A.4}$$

where, as before, s refers to the competitive supply function. When v is independent of the individual aggregation yields H = S(w(1 + v)d). Consistency requires d = D(w)/H or

$$D(w) = S(w(1 + v)d)d, \tag{A.5}$$

defining d = d(w,v) in expectational equilibrium. A comparison with (7) shows that

$$d(w,v) = d(w(1 + v)), \tag{A.6}$$

where d(.) on the right hand side is to be interpreted as the "standard solution" of section 3. We conclude that disutility of work, expressed as a preference for search to work and thus a negative v, affects the rate of unemployment in the same way as a wage reduction does.

Chapter 6

Effective labour supply and optimal wage setting*

1. Introduction

> "... a higher wage benefits every worker as long as the demand elasticity is < 1. ... If the demand elasticity is increased to the point where it is > 1, it will obviously be in everybody's interest to abandon the current wage policy and bring about a decline of the wage level and thus larger average earnings."

In this way the Swedish economist Gösta Bagge summarized part of his findings on the optimal wage in chapter 6 of his seminal work on wage regulation through unions (1917 p. 381). The passage serves as a succinct statement of the main result of the present chapter as well. We note in particular that "average earnings" in Bagge's thesis explicitly refer to an average over periods including unemployment periods (with zero earnings). Thus workers do have a "disequilibrium awareness" (Fisher 1983) in Bagge's analysis. When choosing between jobs they take into account differing unemployment risks as well as differing wage rates.

Moreover, Bagge explicitly emphasized that the wage bill maximizing wage benefits every worker, including newcomers, *as long as rationing is proportional*. His discussion of rationing is remarkably detailed. Having emphasized the general importance of the method of job rationing for the effects of a wage increase on earnings and unemployment (p. 332) he proceeds by differentiating between uniform rationing ("fördelningsmetoden") and concentrated rationing ("koncentreringsmetoden"). Uniform rationing (i.e. proportional rationing) can be effected either through work-sharing or "by finding sometimes one worker sometimes another unemployed" (p. 338). Important examples of work-sharing in Swedish industry in the beginning of this century are cited (pp. 333-336).

In a system with work-sharing a worker's employment is $D(w)h_i/H$, where $D(w)$ denotes the industry's demand for labour at the current wage w, h_i individual labour supply and H aggregate labour supply, $\Sigma_i h_i$. Taking labour supply as exogenously given it follows immediately that every worker's desire to maximize his income $wD(w)h_i/H$ is equivalent to maximizing the wage bill $wD(w)$ for $w \geq w^c$, where w^c denotes the market-clearing wage. Then every worker will prefer w^c if demand is elastic at w^c and otherwise w^o, defined implicitly by $\eta(w^o) = 1$, assuming, as we do for simplicity throughout the present chapter, that the demand elasticity

$\eta(w) = -wD'(w)/D(w)$ is an increasing function of w.

But this argument presupposes rationing by a *visible* hand (work-sharing). In section 2 we shall see that rationing by the *invisible* hand yields the same result – as Bagge conjectured – provided that workers have a purely instrumental attitude towards time spent in market activities, so that the division of this time between work and job search does not matter in itself, only the total payoff in terms of goods.

An immediate corollary is that the classical maximand in the economic theory of trade union behaviour – the wage bill – does have a micro-economic rationale. The present approach to the analysis of trade union behaviour is related to some classical works in section 3 and to recent research in section 4. In section 5 our approach is extended to non-cooperative behaviour. We find that a market-clearing wage is supported by independent workers' self-interest in a market with decentralized wage-setting if *and only if* labour demand is elastic at the market-clearing wage.

2. *The optimal wage*

We assume that the risk for unemployment is equally distributed among workers in a homogeneous and dynamic labour market. Currently employed cannot ignore the risk for future unemployment, nor can currently unemployed ignore the chance of future employment. Formally we assume that every worker expects his employment r_i to be proportional to his supply h_i, $r_i = dh_i$, where d denotes the expected degree of aggregate employment. Note carefully that rationing in proportion to supply in a homogenoeus market presupposes that supply is actually "brought to the market-place", implying in this context that h_i denotes time spent either working or looking for work.

Taking d as well as w as given a worker's supply h_i is given by the solution to the problem

$$\max_{h} u_i(wdh, k - h, dh, h - dh), \tag{1}$$

where u_i denotes his utility index and $k - h$ nonmarket time, devoted to homework or leisure. With a purely *instrumental* attitude towards time spent in market activities (h), the division of this time between employment (dh) and job search ($h - dh$) does not matter in itself, only the total payoff in terms of goods (wdh). Assuming this to be the case (1) reduces to

$$\max_{h} u_i(wdh, k - h). \tag{2}$$

It follows immediately that

$$h_i = s_i(wd) \tag{3}$$

where s_i defines an individual's usual (competitive) supply, $s_i = s_i(w)$, derived on the hypothetical assumption that everything will be sold ($d = 1$).

Aggregation yields $H = S(wd)$. *Consistency* requires $d = D(w)/H$ or

$$D(w) = S(wd)d. \tag{4}$$

It follows that the rate of unemployment *in expectational equilibrium* is not $1 - D(w)/S(w)$ but $1 - d$, where $d = d(w)$ is defined implicitly by equation (4). *Effective* labour supply is not $S(w)$ but $S(wd(w))$. Taking the aggregate demand and supply curves as given (known), we may interpret $1 - d(w)$ as *rational expectations* of the rate of unemployment.

In equilibrium at w a worker's "effective wage" will be $wd(w)$. Multiplying (4) with w we obtain

$$wD(w) = S(wd(w))wd(w). \tag{5}$$

Provided that the supply elasticity is greater than minus one – which we assume to be the case – it follows from (5) that

$$wd(w) \text{ increases if and only if } wD(w) \text{ increases.} \tag{6}$$

In other words, maximization of a worker's average earnings per "market hour" $wd(w)$ is equivalent to maximization of the workers' wage bill $wD(w)$. (And maximization of $wd(w)$ is equivalent to maximization of a worker's indirect utility function $v_i = v_i(w)$, as an application of the envelope theorem to (2) with $d = d(w)$ immediately tells us.) An immediate corollary is that *every* member of a trade union will vote for that wage which maximizes the wage bill $wD(w)$ for $w \geq w^c$.

3. *On the classical maximand in the economics of trade unions*

Gösta Bagge (1917) clearly formulated the hypothesis that given proportional rationing *every* worker in the market – including currently unemployed – would prefer that wage for which the demand elasticity equals one when demand is inelastic at the market-clearing level. Now, does "every" worker include every worker encompassed by the *ordinary* labour supply curve? Bagge apparently took this for granted without even posing the question. Perhaps John Dunlop was the first to realize that there is an analytical problem here, by asking *whose* wage bill is maximized (1966 pp. 4,43).

Dunlop also observed (on p. 44) that "there are hardly any bars to entry in the large industrial-type unions" and then concludes that "the most suitable generalized model of the trade union for analytical purposes is probably that which depicts the maximization of the wage bill for the total membership". But by introducing the union as the basic decision making unit, and replacing individual preferences with union preferences

(pp. 29-32), Dunlop introduced concepts and complications that are difficult to handle.

It is, basically, not the presence of a union but the presence of an *unemployment awareness* that renders "the supply conditions of customary theory ... obscure" (Dunlop 1966 p. 29). Consider a local labour market with inelastic demand and an almost perfectly elastic (almost horizontal) *ordinary* labour supply curve reflecting almost perfect mobility. Thus a great many workers would be attracted to the market by a wage which "maximizes the wage bill" (other things being equal) *if they believed that they would always be fully employed*. But such a belief is not consistent with expectational equilibrium. With actual supply corresponding to the competitive supply curve realized average earnings would be drastically reduced by the high unemployment rate, and since rational workers respond to average earnings and not the wage rate, an inflow of new workers implied by the ordinary supply curve would immediately be reversed.

While Dunlop substituted a vaguely defined "membership function" (1966 pp. 33-34) for the competitive labour supply curve, we have substituted the effective labour supply curve (cf. section 2). Note, in particular, that effective labour supply S(wd(w)) is perfectly *inelastic* at the earnings-maximizing wage w^o (when $w^o > w^c$), even if the competitive supply curve is almost perfectly *elastic* (cf. figure 1).

Dunlop took it for granted that if an enterprise attempts to maximize profits, a trade union should attempt to maximize wages (1966 p. 45). When attempting to formalize this idea, however, Dunlop found the concept somewhat ambiguous and suggested several "conceivable objectives". One of these was maximization of "average wage income for each unit of labour affiliated with the union", which, according to Dunlop (1966 p. 40), "on *a priori* grounds might be regarded as fairly typical". We note that this is, fundamentally, the same notion as maximization of average earnings (cf. Bagge). Now Dunlop observed that this goal is equivalent to that of securing the maximum wage bill if *and only if* the membership function is invariate. But in general, with an upward sloping membership function, that wage which maximizes average wage income may be somewhat lower than that wage which maximizes the wage bill. Hence these two objectives are in general different within Dunlop's framework. Replacing the membership function with the effective supply function, however, we have found that these two objectives are, in fact, equivalent.

4. *Effective labour supply and recent research on trade union behaviour*

Dunlop's suggestion to add wage bill maximization to profit maximization in the economist's tool-box was not accepted by the economics

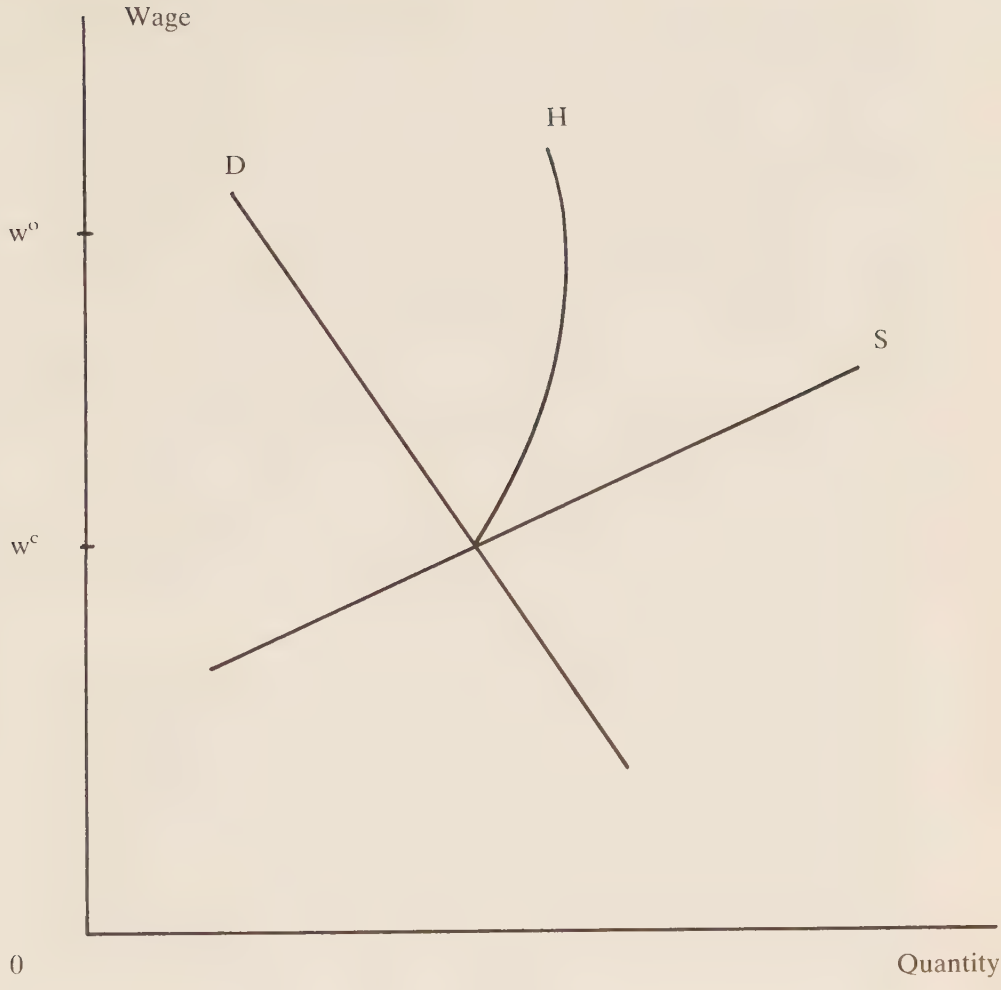

Figure 1. Effective labour supply (H) when demand is inelastic at the market-clearing wage.

profession. Instead his list of "conceivable objectives" might well have added to the rather agnostic view on trade union behaviour that has prevailed until recently. On one hand the vast empirical literature on the union-nonunion relative wage differential – cf. the surveys by Johnson (1975), Parsley (1980) and Lewis (1983) – suggests the implicit presumption that trade unions do, after all, attempt to maximize earnings. But on the other hand this wage goal has proved to be difficult to rigorously define and rationalize, as emphasized not least by Freeman and Medoff (1984 p. 6): "Despite decades in which unions have been part of the

economic scene, economists lack an accepted maximizing theory of union behaviour that would predict the results of bargaining within the union over wage goals."

The recent upsurge of research on trade union behaviour is sometimes claimed to have changed this situation (cf. the survey by Oswald 1985), but in what direction is not yet quite clear (cf. the survey by Pencavel 1985). Moreover, the classical notion of maximizing earnings now appear as a special case only of more general utility functions (cf. Oswald 1985) unless explicitly questioned (cf. Pencavel 1985). The purpose of the present section is to see how the general drift of this modern discussion might be affected by the introduction of the concept of effective labour supply.

In the prevalent approach to the economics of trade unions, a trade union is characterized as a monopoly. Since a monopolist does not have a supply curve, the analysis of union behaviour in terms of "supply and demand" is excluded from the outset. Dunlop (1966) attempted to compensate for this loss by introducing the concept of a "membership function". Some attempts to allow for variable membership have been made recently (cf. section 4 in Oswald's survey), but in most of the modern literature union membership (m) is taken as exogenously given.

Now, to endogenize union membership and at the same time bring labour supply back to the analysis we postulate an open union, with free entry. Then membership is identical to supply – but not *competitive* supply. In expectational equilibrium supply from workers with an unemployment awareness is equal to *effective* labour supply. We have seen that this concept can be derived from the ordinary supply and demand curves (section 2). We have also seen (cf. figure 1) that *effective* labour supply is almost perfectly inelastic up to the income-maximizing wage rate. This means that the potential inflow of workers attracted by an increase of the wage rate w is arrested by the realization that average earnings per hour are not w but (approximately) $w(1 - n/m)$, where n denotes employment (set by firms). There are barriers to entry in our model, but they are endogenous, given by workers' unemployment awareness. This is a choice-theoretic rationale for the conventional presumption in recent research that union membership m is fixed (independent of the wage rate). We also note that in our framework m is simply determined by the intersection between the ordinary demand and supply curves.

In his survey of recent research Oswald (1985) also finds that writers now normally postulate *either* a general quasi-concave union utility function (with wage and employment as arguments) *or* an expected-utility (or utilitarian) function. My approach is obviously akin to the individualistic expected-utility approach, which, as emphasized by Pencavel (1985), presupposes both that all union members stand the same chance of being employed and that all workers know the employment probability. Thus the employment lottery is "fair" and workers have an "unemployment awareness", as in our model.

But uncertainty and the expected-utility hypothesis are not basic to our approach, nor the distinction between employed and unemployed, fundamentally because the "market period" is longer, at least one year. In fact the period considered is so long that the distinction between employed and unemployed is blurred. In a dynamic labour market where employment is set by firms neither unemployed nor employed can identify future labour force status with the current state. Introducing a partial life-cycle perspective employed and unemployed are in fact in the same position, since average earnings will be the same if the period considered is long enough. This might be construed as intra-union income redistribution through work-sharing administered by the law of large numbers, to relate our discussion to that of Pencavel's (1985 section 2). Within this framework we also interpret unemployment insurance as a device to solve unexpected *liquidity problems* by smoothing variations of earnings over time.

Formally, the relation between the expected utility approach and the effective supply approach may be depicted as follows. In its simplest form the expected utility maximand can be written

$$U = (n/m)u(w) + (1 - n/m)u(b), \tag{7}$$

where m denotes union membership, n employment, w the real wage, b the level of unemployment benefit and u(.) the representative worker's utility function. Now we first extend the market period so that there is only one state of nature for each member of the union, defined by the effective wage (n/m)w. The maximand then reduces to u(wn(w)/m), or simply wn(w)/m for every worker, *irrespective of his preferences*. Second we make the argument rigorous by endogenizing m. This is done by reintroducing the labour supply decision and postulating an open union. We have found that

$$m = n(w)/d(w) \tag{8}$$

where d = d(w) is determined by the equation

$$n(w) = S(wd)d, \tag{9}$$

and S denotes the ordinary (competitive) aggregate labour supply function.

The expected utility approach yields results which formally depend upon the utility or disutility of employment (included in u(w)) as well as the utility or disutility of unemployment (included in u(b)). But this formulation does not distinguish between job search and nonmarket time (devoted to homework or leisure). "Disutility of work" may affect the optimal wage policy, but only when interpreted as a preference for *job search* to work, not when interpreted as a preference for leisure (or homework) to work. To see this we have to relax the assumption implied by (2), namely that workers have a purely *instrumental* attitude towards

time spent in the market. This is done in appendix 1. We find that disutility of work, expressed as a preference for search as such to work as such, tends to *increase* the optimal wage.

5. *Noncooperative wage setting*

While Bagge (1917 ch. 6) clearly predicted wage-bill-maximization when rationing was proportional, he also, in the last analysis, characterized proportional rationing as a "rather unrealistic abstraction. Instead of sharing with others old workers naturally attempt to exclude new workers and as far as possible concentrate work on themselves." (1917 p. 381) Bagge claims firstly that this creates *concentration of unemployment* to some workers and secondly that these workers will respond by undercutting the ruling wage. Due to competition from unemployed workers the end result will consequently, according to Bagge – and the market-clearing paradigm – be a return to that wage which clears the market.

But in fact this story is not tenable. To see why, we first make its implicit assumptions explicit:

A.1 Old workers can stop employers from hiring new workers at the ruling market wage.
A.2 New workers are attracted to the market in spite of the high unemployment risk concentrated upon them.
A.3 Newcomers faced with long-term unemployment respond at first by undercutting the ruling wage, not by leaving the market.
A.4 At last, when the wage has reached the competitive level, newcomers do leave the market.

Thus spelled out the plot of the market-clearing story does not make sense, nor the details. We observe, in particular,

1) that A.3 is inconsistent with the prospect of A.4;
2) that wage cutting in A.3 presupposes that barriers to entry – postulated in A.1 – are not, after all, very effective;
3) that A.2 ignores the fact (so much emphasized by Bagge elsewhere) that workers with a "disequilibrium awareness" respond to *average earnings*, not merely the wage rate;
4) that A.1 is incompatible with the rules of the market game, since employment decisions are up to employers (and not old workers) in a market economy.

We turn now to a formal reappraisal of the market-clearing paradigm. Suppose that (reservation) wages w_i are set by workers without cooperation or communication, as in, for instance, Gordon and Hynes (1970) and Futia (1977). According to the market-clearing paradigm independent workers cannot select their own reservation wages without being forced down to the competitive level (w^c). The well-known argument (following

Bertrand) is that $w_i = w > w^c$ cannot define an equilibrium since every worker can increase his employment and income discontinuously by choosing a wage slightly less than w. More precisely $w_i = w$ is not optimal against $w_j = w$, $j \neq i$, for any $w > w^c$, while $w_i = w^c$ is optimal against $w_j = w^c$, $j \neq i$.

It is not always appreciated, however, that the Bertrand story crucially depends on reservation wages being fixed to their initial values. Only in this case can a worker's wage policy be represented by a single number. This is not possible in an industry where workers can observe each other's wages (through wage statistics) and are free to revise their reservation wages at any time t. For then a worker's wage policy consists of 1) an initial wage w_i and 2) a wage response function f_i, characterizing his reactions to other workers' wage behaviour. A non-cooperative equilibrium in this context is an equilibrium in strategies σ_i comprising wage response functions as well as initial reservation wages, $\sigma_i = (w_i, f_i)$.

Of course σ (with components σ_i) cannot be an equilibrium point for $w_i = w > w^c$ with no-response functions $w_i(t) = w_i$, $t > 0$, corresponding to the wage taking hypothesis. But it does not follow from this argument that it will be rational for i to initiate a competitive process leading to w^c. The correct conclusion is that a worker's decision theory cannot be in equilibrium (cf. e.g. Luce and Raiffa 1957 p. 63). In particular his predictions of other workers' behaviour must be revised.

Suppose, to begin with, that the wage taking hypothesis is replaced by the wage cutting hypothesis, corresponding to the wage-response functions

$$w_i = \max \{ w^c, \min \{ w_j \} - \varepsilon \} \tag{10}$$

for some small $\varepsilon > 0$. On observing wages $\{ w_j \}$ a wage cutter consequently reacts by slightly underbidding the lowest bidder. Assuming perfectly flexible wages, so that response times can be ignored, wage strategies with response functions according to (10) are obviously in equilibrium. In fact a worker confronted with wage cutters finds that the resultant market wage will always be w^c, irrespective of his own strategy, so he can do no better than being a wage cutter himself. But we cannot conclude from this argument either that the equilibrium wage must be competitive. For equilibrium is not unique.

To see this we introduce the "kinked" response function

$$w_i = \max \{ w^c, \min \{ w_j \} \} , \tag{11}$$

– or $\min w_j$ for short – and consider the wage strategies

$$\sigma_i^k(w) = (w, \min w_j), \tag{12}$$

implying that every worker asks for w initially and then behaves according to the kinked response function (matches every wage cut instantane-

ously). Let $\sigma^k(w)$ denote the corresponding decision point. It is easy to see that

$$\sigma^k(w) \text{ is an equilibrium point for } w > w^c \text{ if labour demand is inelastic at } w. \quad (13)$$

For then a wage cut $w_i < w$ – taking other workers' *complete* strategies as given – would entail the market wage w_i – and not w – due to the kinked response functions. Hence the effective wage would be $w_i d(w_i)$, which is less than $wd(w)$ when $D(w)$ is inelastic at w, according to (6).

When $w^o > w^c$ – where w^o is defined implicitly by $\eta(w^o) = 1$ – it follows that every w between w^c and w^o is an equilibrium wage. This lack of uniqueness suggests that it might be difficult for workers to reach tacit collusion without communication. But note that given $w_j = w < w^o$, $j \neq i$, $w_i = w^o$ gives the same outcome to i as $w_i = w$. In other words, when wages are perfectly flexible a worker has nothing to lose by asking w^o initially, since he can always match a wage cut. Realizing this every worker should ask w^o, yielding higher average earnings to every worker than any other market wage.

As emphasized by Yellen (1984 p. 204), "it has been widely observed that the existence of excess labour supply does not lead to aggressive wage cutting by workers and firms." While the efficiency-wage models surveyed by Yellen attempt to explain why *firms* avoid cutting wages, the present section serves to explain why *workers* – unemployed as well as employed – may find wage cutting irrational.

The basic prerequisite is that labour demand is inelastic. In that case wage cuts would reduce average earnings for every worker in the market. We have been able to verify this classical conjecture by explicitly modelling workers' labour supply adjustments when there is unemployment. Average earnings refer to a period which is long enough (probably at least one year) to warrant the assumption of proportional rationing in a homogeneous and dynamic labour market where employment is set by firms.

A self-interested worker's reservation wage will be identical to that wage which maximizes the wage bill. When all workers rely on the same estimates of market conditions we also note that "the law of one wage" follows automatically; there is no need to postulate, for instance, that wage differentials are excluded by "workers' morale" (cf. Negishi 1979 p. 92).

In expectational disequilibrium different reservation wages would be interpreted as due to different estimates of w^c and w^o. In equilibrium these estimates should coincide, assuming free entry to the truth. Moreover, if self-interested individuals accept one particular individual's information on market conditions as superior, they should accept this individual as a "barometric" wage leader, a concept suggested by Stigler (1968 p. 228). Within the present frame-work we consequently interpret a trade union not as a "monopolist" but as a *statistician*, supplying infor-

mation on the relevant market conditions to its members (assumed to comprise all workers in the industry).

We conclude that market-clearing in an economy with decentralized wage making does not hinge on competition but on substitutability. Self-interested workers' wage claims are restricted not by competition between workers but by firms ability to substitute other factors of production for labour, as measured by the elasticity of the labour demand curve. Given that firms set employment unilaterally, a market-clearing wage is supported by workers' self-interest *if and only if* labour demand is elastic at the market-clearing wage. If demand is inelastic the equilibrium wage (w^o) implies excess supply. The corresponding "natural" rate of unemployment is, according to appendix 2, approximately equal to $(1 - \eta(w^c))/2$.

Appendix 1.
The optimal wage in the general case

So far we have postulated that a worker has a purely *instrumental* attitude towards time spent in market activities (h), so that the division of this time between work (dh) and search (h − dh) does not matter in itself, only the payoff in terms of goods (wdh). In general, however, an individual's problem is (1) in section 2, i.e., suppressing the last argument of the utility index for simplicity and without restriction,

$$\begin{aligned} &\max_{w,h} u_i(wdh, k - h, dh) \\ &\text{s.t. } w \geq w^c \end{aligned}$$

where $d = d(w)$ is to be interpreted as aggregate demand per unit of aggregate supply *in expectational equilibrium* (to be derived below). The first-order conditions are

$$u_1 h(wd)' + u_3 hd' = 0 \tag{1}$$
$$u_1 wd - u_2 + u_3 d = 0 \tag{2}$$

where u_j denotes the derivative of the utility function u (suppressing the index i) with respect to its j:th argument and $'$ indicates differentiation with respect to w. When $u_3 = 0$ the optimal wage is obviously given by $(wd)' = 0$ (when wd(w) is increasing at $w = w^c$) and the optimal supply by $u_2/u_1 = wd(w)$, as in section 2. But when $u_3 \neq 0$ the problem is no longer separable and a simple explicit solution no longer possible. To get a flavour of its general character we first rewrite (2) as

$$u_2/u_1 = w(1 + v)d, \tag{3}$$

where $vw = u_3/u_1$. We interpret u_3/u_1 as the (marginal) value (in terms of goods) of transferring one hour from search to work *in addition to* the real wage w. In general $v = v(w,h)$ but suppose:

1) that v is approximately constant in the neighbourhood of (w_0,h_0), corresponding to the reference case $u_3 = 0$, $v = v(w_0,h_0)$, and
2) that v is the same for every individual.

Taking v as a constant it follows from (3) that

$$h = s(w(1 + v)d), \tag{4}$$

where s, as before, refers to the competitive supply function. When v is independent of the individual, aggregation yields

$$H = S(w(1 + v)d).$$

Consistency requires $d = D(w)/H$ or

$$D(w) = S(w(1 + v)d)d, \tag{5}$$

which defines $d = d(w)$ in expectational equilibrium. Note that (5) is a simple extension of (4) in section 2.

We now return to (1), which may be written as

$$(d/w)(wu_1(1 - \eta_d) - u_3\eta_d) = 0, \tag{6}$$

where $\eta_d = \eta_d(w)$ denotes the elasticity of d(w) at w. This is not the same as the demand elasticity, $\eta_D = -wD'/D$, but almost so. In fact differentiation of (5), multiplied by w, yields, after some calculation,

$$(1 - \eta_D(w)) = (1 + \eta_S)(1 - \eta_d(w)), \tag{7}$$

where η_S denotes the supply elasticity evaluated at $w(1 + v)d(w)$. Now we take the supply elasticity to be a constant too,

$$\eta_S = \eta_S(w_0(1 + v)d(w_0)),$$

where w_0 refers to our reference case ($u_3 = 0$). It follows from (6) and (7) that the optimal wage, if greater than the competitive wage, is no longer given by $\eta_D(w) = 1$ but by

$$\eta_D(w) = 1 - (1 + \eta_S)v/(1 + v). \tag{8}$$

To interpret (8) we first observe that a preference for work as such to search as such (i.e. $v > 0$) tends to *reduce* a noncompetitive wage. In fact the wage will be competitive if and only if

$$1 - (1 + \eta_S)v/(1 + v) \leq \eta_D(c),$$

i.e. if and only if

$$v \geq (1 - \eta_D(c))/(\eta_S + \eta_D(c)). \tag{9}$$

We conclude that a competitive market wage is supported by workers' self-interest if work is more fun than search to the extent given by (9). On the other hand disutility of work, expressed as a preference for search to work and thus a negative v, tends to *increase* a noncompetitive wage.

Appendix 2.
The natural rate of unemployment

We have seen that the market wage in equilibrium is equal to the market-clearing wage if *and only if* $\eta(w^c) \geq 1$. If $\eta(w^c) < 1$ the equilibrium wage implies excess supply. In this appendix we shall derive a suggestive (approximate) formula for the corresponding "natural" rate of unemployment.

Consider linear approximations of the demand and supply schedules in the neighbourhood of full employment:

$$D = q - a(w - w^c),\ S = q + b(w - w^c). \tag{1}$$

We assume that $b \geq 0$. Let w^o maximize $wD(w)$. We then find that

$$w^o = (q/a + w^c)/2,\ D(w^o) = (q + aw^c)/2 = aw^o, \tag{2}$$

and assume that $\eta(w^c) = aw^c/q \leq 1$ (implying $w^o \geq w^c$).

The corresponding rate of unemployment in expectational equilibrium is determined by (4) in section 2, which with linear approximations and $D(w^o) = aw^o$ according to (2) reduces to

$$aw^o = (q + b(w^o d - w^c))d.$$

Substituting $q = 2aw^o - aw^c$ and dividing by aw^o we obtain

$$1 = d(2 - \gamma + \beta(d - \gamma)), \tag{3}$$

where $\beta = b/a \geq 0$ and $\gamma = w^c/w^o \leq 1$.

When supply is perfectly inelastic ($\beta = 0$) we find (after some calculation) that

$$d_0 = 1/(2 - \gamma) = (1 + \eta(w^c))/2. \tag{4}$$

When $\beta > 0$ the exact solution is

$$d = (-\delta + (\delta^2 + 4\beta)^{1/2})/2\beta, \tag{5}$$

where $\delta = 2 - \gamma(1 + \beta)$, but this expression is not very informative.

Instead we first observe, comparing effective supply when $\beta > 0$ with effective supply when $\beta = 0$, that

$$d(w^o) \leq d_0, \tag{6}$$

since $H(w) = S(wd(w))$ is increasing up to $w = w^o$ when $\beta > 0$. On the other hand (3) and (6) imply

$$d(w^o) \geq 1/(2 - \gamma + \beta(d_0 - \gamma)) = d_0/(1 + \beta d_0^2(1 - \gamma)^2)$$

so that

$$0 \leq (1 - d(w^o)) - (1 - d_0) \leq \beta d_0^3(1 - \gamma)^2. \tag{7}$$

It follows that the natural rate of unemployment $1 - d(w^o)$ is practically independent of the supply elasticity (here measured by $\beta = b/a$) and

approximately equal to $1 - d_0$ for reasonable values of γ and β. Note, for instance, that the error is less than $0.0075\ \beta$ when $\gamma = w^c/w^o = 0.9$ and $1 - d_0 = 0.0909$. Excluding extreme cases we may then safely assume that the natural rate of unemployment can be calculated *as if* labour supply is perfectly inelastic. It then follows from (4) that it is approximately equal to $(1 - \eta(w^c))/2$.

Notes

Chapter 1 On decentralized price setting

* I wish to thank Jan-Erik Björk, Anders Björklund, Kari Lantto, Hans Larsson, Hans Lind, Gösta Rehn, Claes-Henric Siven, Lars Werin and, in particular, Anders Borglin, Lars E.O. Svensson, Eskil Wadensjö and Jörgen W. Weibull for useful comments on earlier drafts.

1 But "there is a large class of economies which have unstable equilibria under the most popular form of the price mechanism." (Hahn 1982 p. 745). Moreover, while for instance Fisher's recent (1983) model shows stability, "it is silent on the question of whether or not the equilibrium asymptotically attained is Walrasian." (1983 p. 156). This conclusion deserves attention not least because it is drawn by an economist previously committed to telling disequilibrium stories with competitive endings (see e.g. Fisher 1973).

2 Cf. Friedman (1983 ch. 3).

3 Cournot equilibrium, Nash equilibrium, Cournot-Nash equilibrium, noncooperative equilibrium and equilibrium point are used as synonyms in what follows.

4 As most vividly exemplified by Hotelling's seminal 1929 paper. It has recently been demonstrated by d'Aspremont, Jaskold Gabszewicz and Thisse (1979) that Hotelling's celebrated Principle of Minimum Differentiation is based upon a non-existent Nash equilibrium. Moreover, if Hotelling's model is modified so as to guarantee existence the principle is reversed: there is, in fact, a tendency for sellers to *maximize* their differentiation.

5 Ignoring income effects in our partial equilibrium setting, i.e. assuming that the consumer's marginal utility of income λ is constant (independent of prices p_j), we might just as well put $\lambda = 1$.

6 Applications of this result must, however, be restricted to circumstances where a single-period (one-shot) model is relevant, i.e. when enterprises have short time horizons or high future discount rates, as emphasized by Scherer (1980 pp. 220-224).

7 See also, for instance, Negishi (1960-61) and Hart (1985).

8 See also, for instance, Axell (1977) and Albrecht, Axell and Lang (1986).

9 Modern "disequilibrium" literature (on fixprice equilibria with quantity rationing) is no exception; see for example the monographs by Barro and Grossman (1976), Malinvaud (1977), Casson (1981), Benassy (1982) and Cuddington, Johansson and Löfgren (1984).

Chapter 2 Noncooperative flexible pricing in a homogeneous market

* The basic ideas underlying this note were independently developed by each of the two authors. By Farm in the context of "rational price-setting" under proportional rationing (Farm (1985a, 1986)), and by Weibull in the context of "price-cut matching strategies" of neo-Keynesian firms (Weibull (1984)). We are grateful for helpful comments from Henrik Horn, Lars E.O. Svensson and Hans Wijkander, as well as from the participants of a seminar at the Institute for International Economic Studies.

1 Extensions of the present model to include production will be discussed in subsequent chapters and Weibull (1986).

2 Stigler (1964) and Spence (1978) analyze the difficulties of tacit collusion in the presence of imperfect information (in particular, the possibility of "secret price cutting") and product heterogeneity.

3 See Green (1980) or Weinrich (1984) for rigorous derivations of proportional stochastic rationing from basic axioms concerning general properties of the underlying rationing scheme.

4 However, it does not follows that p^c is a NE in all situations. For if some individual contingent demand function is inelastic at p^c, then a price rise is profitable. As emphasized by Shubik (1959, ch.5), the derivation of contingent demand curves is, in general, a complex problem though.

5 Also Marschak and Selten (1978) assume that reaction times are negligible, so that transitory profits during the price revision process can be ignored. However, in contrast to the present model, they use an equilibrium concept which "is not an equilibrium action n-tuple in the traditional Nash-Cournot ... sense, ..." (op.cit. p.73).

6 A further extension is given in Weibull (1986), in which tatonnement pricing is modelled as an infinite game in extensive form with perfect recall.

7 If $p^o > p^c$, then $p^e = p^oD(p^o)/Q < p^oD(p^c)/Q = p^o$. By assumption p^o maximizes $pD(p)$, thus $p^e = p^oD(p^o)/Q > p^cD(p^c)/Q = p^c$.

8 It is not sufficient to use the simpler response function $f^m(p) = \min \{ p_i \}$. For if all sellers but one would have such strategies, then it could be advantageous for the remaining seller i to (1) quote a very low price, e.g. $p_i(0) = 0$, hence driving all other sellers down to that price, and (2) have a response function f_i with the property $f_i(p_i(0),..,p_i(0)) > p_i(0)$. Such a strategy could lead to a favourable "contingent demand" for seller i, because of the small income loss incurred at purchases from other sellers.

9 A result similar in spirit, but different in its theoretical foundation, is given in Osborne (1976). He shows that cartels are internally stable once the contract surface (Pareto frontier) has been located and cheating can be detected. Green and Porter (1984) extend this analysis of cartel self-enforcement to the case of demand uncertainty.

10 If each vector σ^i supports p^o as a uniform equilibrium price, then $f^i(p^o,..,p^o) = (p^o,..,p^o)$ for every i. Hence, if $\sigma^{mix} = (\sigma_1^1,..,\sigma_n^n)$, then $p_i^{mix} = p^o$ and $f_i^{mix}(p^o,..,p^o) = p^o$ for every component i of σ^{mix}.

11 As recently emphasized in the literature on oligopoly theory (cf. e.g. Bresnahan (1981)), reaction functions should be consistent. Interpreting "consistent conjectures" concerning other agents' behaviour as rational expectations of their (complete) market strategies, one may note that the strategies singled out by the Nash equilibrium condition are consistent in precisely this way, cf. Johansen (1982) and Friedman (1983, p.110).

Chapter 3 Oligopolistic pricing

* This chapter – which was presented at the First Annual Congress of the European Economic Association in Vienna, August 1986 – is a revised version of Farm (1986). – I wish to thank Anders Borglin, Nils Gottfries, Henrik Horn, Lars E.O. Svensson, Eskil Wadensjö and Jörgen W. Weibull for useful comments on earlier drafts.

1 Production to orders – when sales precede production – is discussed in chapter 1.

2 Like a systematic underexposure of the products from one particular firm in every shop.

3 (30) is obvious while (31) follows from

$$\pi = pdq_i - cq_i = pd(1 - c/pd)q_i = pd(1/n)(1/n)D(p)/d$$

according to (21).

4 Recall that demand is elastic at the monopoly price ($\eta(p^m) > 1$) so $pD(p)$ must be decreasing at p^m and hence $p^o < p^m$.

5 Substituting $p^u = cn/(n - 1)$ in the condition $p^m \leq p^u$ we obtain $n \leq p^m/(p^m - c)$. Since p^m is defined implicitly by the equation

$$p^m = \frac{c}{1 - 1/\eta(p^m)}$$

it follows that $p^m \leq p^u$ is equivalent to $n \leq \eta(p^m)$.

6 Cf. Shubik (1959 pp. 111, 82-91).

7 Nonexistence of an equilibrium price (often interpreted as a fluctuating market price) in situations *with capacity constraints* was first observed by Edgeworth (1925 p. 120). See also Shubik (1959 p. 92) and Shapley and Shubik (1969).

8 Cf. Friedman (1983 pp. 66, 124). – Also note that the equilibrium concept in Marschak and Selten (1978) is explicitly claimed to be new and thus distinct from the Cournot-Nash equilibrium. The present chapter argues instead that what is needed in oligopoly theory is not a new equilibrium concept but new representations of market strategies.

9 As recently emphasized by e.g. Bresnahan (1981) reaction functions cannot be taken as given in oligopoly theory: they must be *consistent*. But as noted by Friedman (1983 p. 110) this insight is not new. In fact we interpret "consistent conjectures" of rivals' reaction functions as *rational expectations of their strategies* and recall that the strategies singled out by a Nash equilibrium for other players should be interpreted in exactly this way, as emphasized not least by Johansen (1982).

10 Recall that $p^o < p^m$ since a monopolist always operates where the demand elasticity exceeds unity ($\eta(p^m) > 1$.

11 A further output increase will not be profitable, since, according to (17) with $d = 1$,

$$\delta p_i/\delta q_i = p_i(1 - \alpha_i) - c$$

and $\delta p_i/\delta q_i \leq 0$ at $p_i = p$ and $q_i = q_i^{\sim}(p)$ by construction (Cournot equilibrium) while smaller p_i and greater α_i can only decrease $\delta p_i/\delta q_i$.

Chapter 4 Noncompetitive equilibria in atomistic markets

* This chapter – which was presented at the European Meeting of the Econometric Society in Budapest, September 1986 – is a revised version of Farm (1985 a). Section 6 builds on Farm (1985 b). – I am grateful to Anders Borglin, Lars E.O. Svensson, Eskil Wadensjö, Jörgen W. Weibull and participants of seminars at the Swedish Institute for Social Research for useful comments on earlier drafts.

1 Chamberlin 1962 pp. 104, 106, 203-206.

2 Chamberlin 1962 pp. 56, 69. 204. – For recent contributions to the theory of monopolistic competition see e.g. Dixit and Stiglitz (1977) and Hart (1985).

3 Recall that this literature relies on perfect information on the distribution of quoted prices, making the label "imperfect" information rather dubious. – It appears to me that what distinguishes models of decentralized price-setting from each other are different assumptions about what price-setters know. With perfect information on what agents are supposed to know they will quote the corresponding (noncooperative) equilibrium prices at once. Equilibrium will be instantaneous with perfect information. With imperfect information – i.e. approximate *estimates* of the aggregate demand and supply curves – there will be price adjustment in real time as long as estimates are being revised, but the details of such a learning process are outside the scope of the present study.

4 Cf. Friedman (1983).

5 Since then firm i can always sell all of its competitive supply corresponding to p_i, $q_i = s_i(p_i)$, yielding profits $\pi_i = p_iq_i - c_i(q_i)$. But $\pi_i' = p_i'q_i - c_i(q_i)$ for any $p_i' \leq p^c$ and $\pi_i' > \pi_i$ for any p_i' such that $p_i < p_i' \leq p^c$. It follows that $p_i < p^c$ cannot be an optimal price.

6 Of course every consumer can turn to rivals selling at p if i quotes $p_i > p$ when $p > p^c$, since then $D(p) < Q^e(p)$ and (with atomistic firms) $D(p) < Q^e(p) - q_i^e(p)$. Also note that defections from the kinked response function are – if they matter at all – equivalent to initiating price cuts or price rises at some later time during the market period and hence unprofitable for the same reasons as initial price changes are unattractive.

7 At this point the simplicity of the present approach as compared to that of Iwai (1981) is particularly striking. $d(p^o)$ corresponds to Iwai's g^*, a ratio of demand to supply which is determined by a rather "weird equation ... to which it is hard to give an interpretation" (Iwai 1981 p. 28). Moreover, while g^* may be greater than unity (Iwai 1981 p. 30), $d(p^o)$ cannot be greater than unity, a result which is more akin to Chamberlin's vision of a market economy as one characterized by excess capacity (Chamberlin 1962 pp. 104-109).

8 As an example consider *indivisible supply*. In this case either everything or nothing is sold, i.e. $Pr(r = 0) = 1 - a$ and $Pr(r = q) = a$ for some a, $0 \leq a \leq 1$. Obviously $a = d$ (since $Er = aq = dq$), and the probability distribution of r is completely determined by d.

Chapter 5 Job search and effective labour supply

* This is a revised version of the first few sections in Farm (1985 c) and section 4 in Farm (1985 b). – I would like to thank James Albrecht, Jaime Behar, Bertil Holmlund, Eugenia Kazamaki, Ann-Marie Steen, Lars E.O. Svensson and Eskil Wadensjö for valuable comments on earlier drafts.

1 In a market with friction, as measured by $f = (D - R)/D$ where R denotes aggregate employment $\Sigma_i r_i$, we find that

$$Er_i = (1 - Ef)(D/H)h_i.$$

In deriving this formula (generalizing the proportional rationing scheme to a stochastic setting) we do not *assume*, as Green (1980) and Weinrich (1984) do, that the only parameters affecting the probability distribution of r_i are h_i, D and H. Weinrich generalizes Green's result by *dropping* Green's anonymity

requirement; here we *reformulate* Green's anonymity requirement and make it the basis of our approach to disequilibrium analysis.

2 Note, incidentally, that proportional rationing is but a special case of rationing by a *visible* hand. It corresponds either to reduced working hours for everyone (irrespective of age etc) at every time, or to rationing by lottery (not e.g. seniority) when unemployment takes the form of layoffs (in which case the law of large numbers will ensure proportional rationing in the long run).

3 In fact (8) is true as long as the supply elasticity is greater than minus one. To see this we first write $W = wd(w)$. Differentiating $W = wD(w)/S(W)$ we then find, after some calculation,

$$(w/W)(\delta W/\delta w) = (1 - \eta_D)/(1 + \eta_S),$$

where the elasticities η_D and η_S are evaluated at w and wd(w) respectively.

Chapter 6 Effective labour supply and optimal wage setting

* This is a revised version of the second half of Farm (1985 c). – I would like to thank James Albrecht, Jaime Behar, Bertil Holmlund, Eugenia Kazamaki, Ann-Marie Steen, Lars E.O. Svensson and Eskil Wadensjö for valuable comments on earlier drafts.

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